4 General Properties of Nuclei

4.1 Introduction

The basic properties of nucleons were presented in chapters 1, 2, and 3, together with the development of the deuteron theory. Our purpose in this and the following chapters is to study the physics of nuclei with any number A of nucleons, to establish the systematics of their properties, and to present the theories that aim to explain them. However, the approach we have followed for the deuteron is not applicable here. The Schrödinger equation is already not exactly soluble for a three-nucleon system, and to establish the properties of a heavy nucleus starting from the interaction of all its constituents is not a feasible task. The reasonable approach is the use of idealized models that incorporate part of the physics involved and explain a limited set of experimental data. This chapter presents the general characteristics of nuclei and introduces some basic ideas that will be employed in the elaboration of nuclear models. The detailed presentation of these models will be done in chapter 5.

4.2 Nuclear Radii

The radius of protons and neutrons that compose the nucleus is of the order of 1 fm. Suppose that a nucleus has A nucleons distributed inside a sphere of radius R. If the nucleons could be considered as small hard spheres of radius r in contact with each other, we could write

$$A \cong \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3}$$

or, in another way,

$$R \cong r_0 A^{1/3}, \tag{4.1}$$

where we put r_0 in place of r to take into account that, even in this model of "packed"

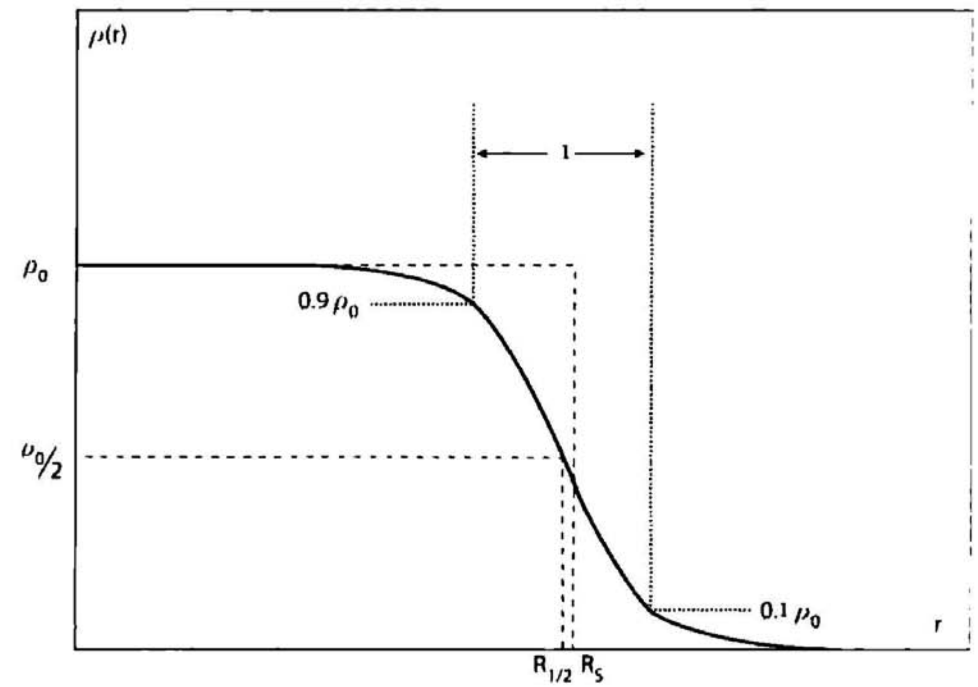

Figure 4.1 Nucleon density as a function of the distance to the center of the nucleus, obeying a typical Fermi distribution.

spheres, there are empty spaces among them, and the nuclear volume should be greater than the simple sum of volumes of each sphere. We expect, therefore, that r_0 is somewhat greater than 1 fm.

We can also infer the radius of a nucleus experimentally. The experiments that give the most precise results are the ones that use electron scattering. The electrons are accelerated and thrown against a target, interacting electromagnetically with the protons and bringing, on their way out, information on how these protons are distributed inside the nuclei. In other words, measurement of electron scattering allows us to deduce the charge distribution in nuclei. If we suppose that the neutron and proton densities have the same distribution shape, then the charge distribution in nuclei will be identical to the mass distribution.

The method used to measure the charge distribution in nuclei was developed mainly by R. Hofstadter and collaborators [Ha56] using the Stanford University linear accelerator. The results from several experiments show that the charge (or mass) density, that is, the number of charges (or mass) per unit volume, can be well described by

$$\rho(r) = \frac{\rho_0}{1 + e^{(r - R_{1/2})/a}}, \tag{4.2}$$

where ρ_0, $R_{1/2}$, and a are fitting parameters. The functional form of r above is known as the *Fermi distribution*. It falls to half its center value at $r = R_{1/2}$ (figure 4.1). Expression (4.2) tell us that the nucleon distribution inside the nucleus is not like a homogeneously occupied sphere with a well-defined radius.

The nuclei have a diffuse surface, with the density decreasing rapidly for $r \gtrsim R_{1/2}$. The quantity a gives the surface diffuseness width. The interval where the density decreases from 90% to 10% from the center value has width $t = 4.4a$. Figure 4.2 shows the charge

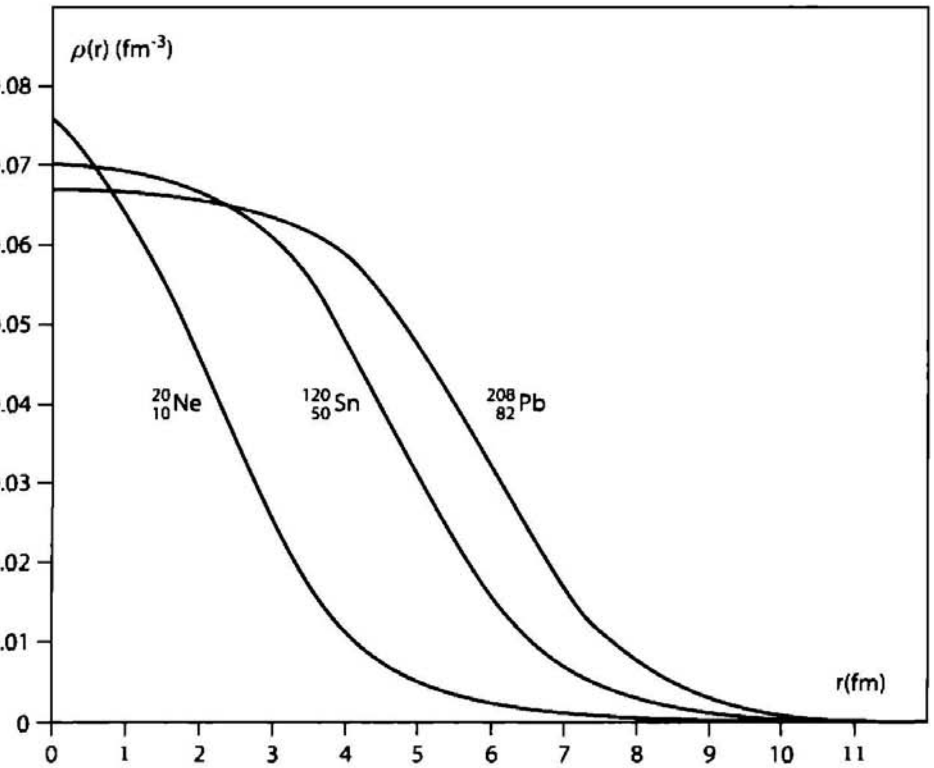

Figure 4.2 Charge distribution in three nuclei, representing light, medium, and heavy elements [Ho57].

distribution in several nuclei, obtained from the analysis of experimental results of Hofstadter and colleagues in 1957 [Ho57]. It gives a good idea of the behavior of this quantity for a large range of masses. An examination of these results shows that the charge distribution of nuclei with $A > 20$ is well described by (4.2), with

$$\rho_0 = 0.17\,\frac{Z}{A}\ \text{fm}^{-3}, \qquad a = 0.54\ \text{fm},$$
$$R_S = 1.128\ A^{1/3}\ \text{fm}, \qquad R_{1/2} = R_S - 0.89A^{-1/3}\ \text{fm}. \tag{4.3}$$

R_S is the radius of a homogeneously charged sphere, with constant charge density ρ_0 and total charge Ze. If we wish to use (4.2) to describe nucleon instead of charge density for $A > 20$, we can use the same values as in (4.3) but with $\rho_0 = 0.17$ nucleons/fm^3. For $A < 20$ the Fermi distribution, (4.2), is not adequate to describe the charge (or nucleon) distribution, since for nuclei with few nucleons the idea of defining a surface is less clear.

Besides electron scattering, other methods are used to determine the nuclear radius experimentally. One of them is to study the *muonic atom*. The muon is the nearest relative of the electron and has mass equal to $207m_e$. Muons can be captured by nuclei and form atoms, where they play the role of the electrons. The atomic levels in a muonic atom are analogous to those of a normal atom; the difference is in the energy of the levels and in the radii of the muonic orbits.

The energy of an atomic level is given (without relativistic corrections) by

$$E = -\frac{\mu Z^2 e^4}{2\hbar^2 n^2}, \tag{4.4}$$

where n is the principal quantum number and

$$\mu = \frac{m_e M}{m_e + M} \tag{4.5}$$

the reduced mass of the atom (M is the nuclear mass). In a muonic atom the value of μ is about 200 times greater and the levels will be more strongly bound. If this were the only difference, the transition energy between two equal levels for normal and muonic atoms (ΔE and $\Delta E'$, respectively) would be given by the ratio $\Delta E/\Delta E' \cong 1/200$. However, since the orbit radius (determined from the Bohr atom model) is given by

$$r = \frac{n^2\hbar^2}{\mu Z e^2}, \tag{4.6}$$

the radius of a muonic atom is 200 times smaller than that of a normal atom. If the muon is in the lowest level (K shell) there will be a reasonable probability of finding it inside the nucleus. The atomic levels of a muonic atom will be modified due to the interaction of the part of the wavefunction of the muon that lies inside the nucleus. Thus, instead of $\Delta E'$, the transition energy to the lowest level will be $\Delta E' + \Delta E_{vol}$, with ΔE_{vol} being given by

$$\Delta E_{vol} = e \int_0^R \left[\psi_2^2(r) - \psi_1^2(r)\right]\left[U_V(r) - U(r)\right] 4\pi r^2\, dr, \tag{4.7}$$

where $\psi_n^2(r)dV$, with $dV = 4\pi r^2 dr$, is the probability density of finding the n orbital muon in the volume dV inside the nucleus. $U(r) = Ze/r$ is the Coulomb potential that the muon would feel if the nucleus were pointlike, and $U_V(r)$ is the realistic potential at point r for a finite size nucleus. For a uniform charge distribution one can show that (R is the nuclear radius)

$$U_V(r) = \frac{Ze}{R}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2\right]. \tag{4.8}$$

The above relations point out that from a measurement of the transition energy between two levels of a muonic atom we can infer the radius R of the corresponding nucleus [En74].

The nuclear radii can also be obtained from a study of nuclear reactions or collisions induced by α-particles and other nuclei. The Rutherford experiments with α-particles around 1911 [Ru11] obtained the value (4.1) with $r_0 \cong 1.2$ fm.

4.3 Binding Energies

For every bound system, the mass of the system is smaller than the sum of the masses of its constituents, if measured separately. This property was presented earlier for the case of the deuteron and is an important attribute of the nucleus for each A value. In this respect nuclear physics is unique, since in other fields of physics the loss of mass corresponding to binding is negligible compared to the mass of the system itself.

The binding energy of a nucleus, which is conceptually the energy needed to separate all the nucleons in the nucleus, is easily calculated if we remember that it should be equal

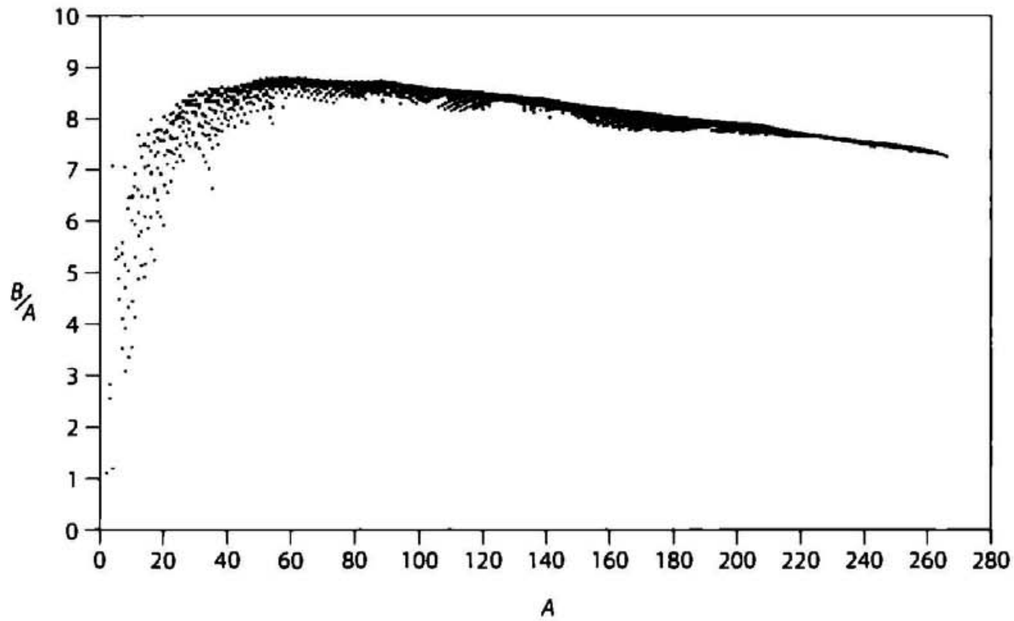

Figure 4.3 Binding energy per nucleon, B/A, as a function of the mass number A.

to the mass loss when the nucleus is formed. For a nucleus ${}^A_Z X$, with proton number Z and neutron number $N = A - Z$, it is given by

$$B(Z, N) = \{Zm_p + Nm_n - m(Z, N)\}c^2, \tag{4.9}$$

where m_p is the proton mass, m_n the neutron mass, and $m(Z, N)$ the mass of the nucleus. These masses can be measured by means of the *mass spectrograph*, an apparatus based on the trajectory that a charged particle describes under the action of an electric and a magnetic field. Since the neutron does not have charge, its mass has to be measured by other processes. We can, for example, measure the deuteron and proton masses and, knowing the deuteron binding energy by means of its dissociation by a photon, deduce the rest mass of the neutron.

The binding energy defined by (4.9) is always positive. In figure 4.3 we show the binding energy per nucleon, B/A, as a function of A, for all known nuclei. The average value of B/A increases quickly with A for light nuclei and decreases slowly from 8.5 MeV to 7.5 MeV beginning with $A \cong 60$, where it has a maximum. We can say that for with nuclei $A > 30$ the binding energy B is approximately proportional to A.

In the light nuclei region, four points are observed whose binding energy per nucleon is greater than the local average: ${}^4_2He$, ${}^8_4Be$, ${}^{12}_6C$, and ${}^{16}_8O$. The nuclei ${}^{20}_{10}Ne$ and ${}^{24}_{12}Mg$ also lie in the upper part of the graph. Notice that these nuclei have equal and even proton and neutron numbers.

The initial rise of the B/A curve indicates that the fusion of two light nuclei produces a nucleus with greater binding energy per nucleon, releasing energy. This is the origin of the energy production in the stars. The initial stage in the evolution of a star is the production of helium by means of hydrogen fusion; in later stages the production of heavier elements occurs by fusion of lighter nuclei. It is not difficult to conclude from figure 4.3 that if a star follows the normal course of its evolution without the occurrence of major incidents (gravitational, etc.) it will end as a cold cluster of nuclei with $A \cong 60$, since from that time on nuclear fusion is no longer energetically advantageous.

On the other side of the maximum binding energy per nucleon, for heavy nuclei the division into approximately equal parts (*nuclear fission*) releases energy. Figure 4.3 shows that in this case the energy gain is nearly 1 MeV per nucleon and thus about 200 MeV is gained in each event. The nuclear fission process is the basis of nuclear reactor operation, where neutrons strike heavy elements (normally uranium or plutonium), leading them to fission and to produce more neutrons, forming chain reactions. It is also the basis of war artifacts. The explosive devices of nuclear origin that receive the somewhat inappropriate denomination atomic bombs have this same character due to the fast but now uncontrolled chain reactions that release enormous amounts of energy in a small volume.

The fact that the binding energy per nucleon is approximately constant for $A > 30$ is due to the *saturation of the nuclear forces*. Each nucleon is bound to $A - 1$ other nucleons, in such a way that there are in total $A(A-1)/2$ nucleon-nucleon bindings in a nucleus with mass number A. Thus, if the range of nucleon-nucleon forces were greater than the nuclear dimension, the binding energy B should be proportional to the number of bindings between them, that is, B should be proportional to A^2. Since this is not the case, one concludes that the nucleon-nucleon forces have a range much smaller than the nuclear radius.

The binding energy B is the energy necessary to separate all the protons and all the neutrons of a nucleus. Another quantity of interest is the *separation energy* of a nucleon from the nucleus. The separation energy of a neutron from a nucleus (Z, N) is given by

$$S_n(Z, N) = [m(Z, N-1) + m_n - m(Z, N)]c^2 = B(Z, N) - B(Z, N-1). \tag{4.10}$$

In the same way we can define the separation energy of a proton or an α-particle. The separation energy can vary from a few MeV to about 20 MeV and depends very much on the structure of the nucleus. One observes that S_n is greater for nuclei with an even number of neutrons. We can define a *pairing energy* as the difference between the separation energy of a nucleus with an even number of neutrons and that of a neighbor nucleus, that is,

$$\delta_n(Z, N) = S_n(Z, N) - S_n(Z, N-1), \tag{4.11}$$

where N is even. One observes experimentally that both δ_n and δ_p are about 2 MeV.

When one plots the separation energy versus Z or N one sees that at the values 2, 8, 20, 50, 82, 126, 184[1] the separation energy changes abruptly. These values are known as *magic numbers*, and nuclei with magic Z (or N) have the last proton (or neutron) shell complete, similar to that which occurs with the closed shells of electrons in noble gases. This subject will be discussed in connection with the shell model for the nucleus.

[1] The last two values refer only to neutrons.