1 Hadrons

1.1 Nucleons

The scattering experiments made by Rutherford in 1911 [Ru11] led him to propose an atomic model in which almost all the mass of the atom was contained in a small region around its center called the *nucleus*. The nucleus should contain all the positive charge of the atom, the rest of the atomic space being filled by the negative electron charges.

Rutherford could, in 1919 [Ru19], by means of the nuclear reaction

$$ {}^{4}_{2}\mathrm{He} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{17}_{8}\mathrm{O} + \mathrm{p}, \tag{1.1} $$

detect the positive charge particles that compose the nucleus called *protons*. The proton, with symbol p, is the nucleus of the hydrogen atom; it has charge $+e$ of the same absolute value as that of the electron, and mass

$$ m_p = 938.271998(38)\ \ \mathrm{MeV}/c^2, \tag{1.2} $$

where the values in parentheses are the errors in the last two digits.

From study of the hydrogen molecule one can infer that the protons in the molecule can be aligned in two different ways. The spins of the two protons can be parallel, as in *orthohydrogen*, or antiparallel, as in *parahydrogen*. Each proton has two possible orientations relative to the spin of the other proton, and like the electron the proton has spin $\frac{1}{2}$.

In orthohydrogen the wavefunction is symmetric with respect to the interchange of the spins of the two protons, since they have the same direction, and experiments show that the wavefunction is antisymmetric with respect to the interchange of the spatial coordinates of the protons. This justifies the wavefunction being antisymmetric with respect to the complete interchange of the protons. In parahydrogen the wavefunction is also antisymmetric with respect to the complete interchange of the two protons, being antisymmetric with respect to the interchange of the spins of the protons and symmetric with respect to the interchange of their spatial coordinates. This shows that the protons obey Fermi-Dirac statistics; they are *fermions* and the Pauli exclusion principle is applicable to them. At most one proton can exist in a given quantum state.

The *neutron*, with symbol n, has charge zero, spin $\frac{1}{2}$, and mass

$$m_n = 939.565330(38)\ \ \text{MeV}/c^2. \tag{1.3}$$

In 1930, Bothe and Becker [BB30] discovered that a very penetrating radiation was released when boron, beryllium, or lithium was bombarded with α-particles. At that time it was thought that this penetrating radiation was γ-rays (high-energy photons). In 1932, Curie and Joliot [CJ32] figured out that the radiation was able to pull out protons from a hydrogen-rich material. They suggested that this was due to Compton scattering, that is, the protons recoiled after scattering the γ-rays. This hypothesis, however, meant that the radiation consisted of extremely energetic γ-rays, and no explanation could be given for the origin of such high energies. Also in 1932, Chadwick [Ch32] showed, by means of an experiment conducted at the Cavendish laboratory in Cambridge, that the protons ejected from the hydrogen-rich material had collided with neutral particles with mass close to the mass of the proton. These were neutrons, the neutral particles that composed the penetrating radiation discovered by Bothe and Becker. The reaction that occurred when beryllium was bombarded with α-particles was

$${}^{4}_{2}\text{He} + {}^{9}_{4}\text{Be} \ \rightarrow\ {}^{12}_{6}\text{C} + {}^{1}_{0}\text{n}. \tag{1.4}$$

The existence of the neutron was also necessary to explain some features of the molecular spectrum showing that the wavefunctions of nitrogen molecules were symmetric with respect to interchange of the two ^{14}N nuclei. As a consequence, the ^{14}N nuclei were *bosons*. This could not be explained if the ^{14}N nucleus were composed only of protons and electrons, since 14 protons and 7 electrons are needed for that, which means an odd number of fermions. A system made up of an odd number of fermions is a fermion, since the interchange of two systems of this type can be made by the interchange of each of their fermions, and each change of two fermions changes the sign of the total wavefunction. In the same way, we can say that a system composed of an even number of fermions is a boson. This shows that if the ^{14}N nucleus is formed by 7 protons and 7 neutrons it is a boson, assuming that the neutron is a fermion. In this way, the study of the N_2 molecule led Heitler and Hertzberg [HH29] to conclude that atomic nuclei are composed of protons and neutrons and not of protons and electrons.

Several other studies established that neutrons obey the Pauli principle and thus are fermions, having spin $\frac{1}{2}$. We recall that particles with fractional spin $(2n+1)/2$ are fermions, and that particles with integer spin are bosons. Protons and neutrons have similar properties in several aspects, and it is convenient to utilize the generic name *nucleon* for both.

1.2 Nuclear Forces

The origin of the Coulomb force between charged particles is the exchange of photons between them. This is represented by the *Feynman diagram* (a) of figure 1.1. In this diagram

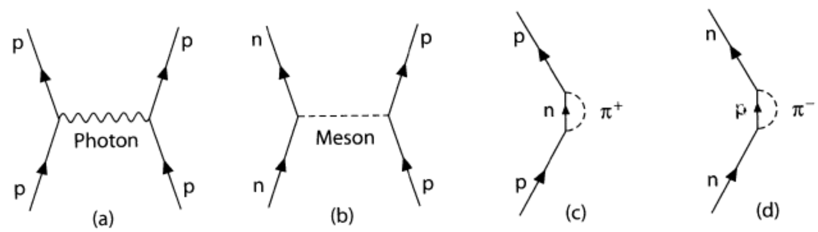

Figure 1.1 Diagrams that represent (a) the electromagnetic interaction, which occurs by exchange of photons, and (b) the nuclear interaction, due to meson exchange. (c and d) Virtual dissociation of the nucleons, giving rise to the anomalous magnetic moment.

lines oriented up represent the direction in which time increases. At some instant of time the particles exchange a photon, which gives rise to attraction or repulsion between them. The photon has zero mass and the Coulomb force is a long-range force.

The force that keeps the nucleus bound is the *nuclear force.* It acts between two nucleons of any type and, in contrast to the Coulomb force, it is of short range. In 1935 Yukawa [Yu35] suggested that the nuclear force has its origin in the exchange of particles with finite rest mass between the nucleons. These particles are called *mesons,* and this situation is described by the Feynman diagram of figure 1.1(b). In the emission of a meson with rest mass M, the total energy of the nucleon-nucleon system is not conserved by the amount $\Delta E = Mc^2$. From Heisenberg's uncertainty principle, $\Delta E\,\Delta t \simeq \hbar$, the exchanged meson can exist during a time Δt (in which violation of energy conservation is allowed), such that

$$\Delta t \simeq \frac{\hbar}{\Delta E} = \frac{\hbar}{Mc^2}. \tag{1.5}$$

During this time the exchanged meson can travel at most a distance

$$R = c\Delta t \simeq \frac{\hbar}{Mc}, \tag{1.6}$$

since the velocity of light c, is the maximum velocity. Then, if the nuclear force can be described by meson exchange, the mesons would exist "virtually" during a time permitted by the uncertainty principle. The nuclear force range would be approximately $\hbar/Mc$. Experimentally one finds that the nuclear force range is $R \simeq 10^{-13}$ cm. Thus, an estimate for the meson mass is

$$M \simeq \frac{\hbar}{Rc} \simeq 0.35 \times 10^{-24}\ \text{g} \simeq 200\ \text{MeV}, \tag{1.7}$$

where $1\ \text{MeV}/c^2 = 1.782 \times 10^{-27}$ g (for brevity, one normally omits c^2).

In 1936, Anderson and Neddermeyer [AN36] observed cosmic rays in a bubble chamber and found a particle with mass approximately equal to that predicted by Yukawa. These particles were investigated during the next ten years but, because their interaction with nucleons was extremely weak, they could not be the Yukawa meson. This puzzle was solved by Lattes, Muirhead, Powell, and Ochialini [La47]. They discovered that there are two types of mesons: the μ-mesons and the π-mesons. The π-meson interacts strongly with nucleons, but has a very short lifetime and decays into a μ-meson, the particle identified previously by Anderson. The *muon*, as it is known today, has a longer lifetime and does not interact strongly with other particles. The muon does not enter into the description of the nuclear force and is classified among the *leptons*, the family of light particles to which the electron belongs.

The π-meson, known as the *pion*, is the particle predicted by Yukawa. Pions were produced in the laboratory for the first time by Gardner and Lattes in 1948 [GL48], using 340 MeV α-particles from the University of California synchrocyclotron.

1.3 Pions

The pion exists in three charge states, π^+, π^0, and π^-. The π^+ and π^- have the same mass, 139.56995(35) MeV, and the same mean lifetime, $\tau = 2.6 \times 10^{-8}$ s, and decay almost exclusively by the process

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \pi^- \to \mu^- + \bar{\nu}_\mu, \tag{1.8}$$

where μ^+, μ^- are the positive and negative muons, ν_μ is the *muonic neutrino*, and $\bar{\nu}_\mu$ is the corresponding antineutrino. Only a small fraction, 1.2×10^{-4}, of pions decay by

$$\pi^+ \to \mathrm{e}^+ + \nu_e, \quad \pi^- \to \mathrm{e}^- + \bar{\nu}_e, \tag{1.9}$$

yielding, respectively, a positron (electron) and an *electron neutrino* (*antineutrino*). (Neutrinos are particles with zero charge and very small mass. Electron neutrinos have a significant role in the β-decay theory; see chapter 8.)

The decay fraction in a given mode is called the *branching ratio*. Charged pions can also decay as

$$\pi^+ \to \mu^+ + \nu_\mu + \gamma, \quad \pi^- \to \mu^- + \bar{\nu}_\mu + \gamma, \tag{1.10}$$

also with a branching ratio of 1.2×10^{-4}.

The mass of the neutral pion π^0 is 134.9764(6) MeV, a value 4.6 MeV smaller than that of the charged pions. π^0 decays as

$$\pi^0 \to \gamma + \gamma, \tag{1.11}$$

with a branching ratio of 98.8 %, by

$$\pi^0 \to \mathrm{e}^+ + \mathrm{e}^- + \gamma, \tag{1.12}$$

with a branching ratio of 1.2 %, and by other much less probable processes. The π^0 total lifetime is $(8.4 \pm 0.6) \times 10^{-17}$ s.

The simplest way to produce pions involves collisions between nucleons:

$$\begin{aligned} &\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \pi^0, \quad &\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{n} + \pi^+, \\ &\mathrm{p} + \mathrm{n} \to \mathrm{p} + \mathrm{p} + \pi^-, \quad &\mathrm{p} + \mathrm{n} \to \mathrm{p} + \mathrm{n} + \pi^0. \end{aligned} \tag{1.13}$$

Pion properties can also be investigated by reactions induced by them, like elastic scattering,

$$\pi^- + \mathrm{p} \to \pi^- + \mathrm{p}, \tag{1.14}$$

inelastic scattering,

$$\pi^- + \mathrm{p} \rightarrow \pi^0 + \pi^- + \mathrm{p}, \tag{1.15}$$

or charge exchange reactions,

$$\pi^- + \mathrm{p} \rightarrow \pi^0 + \mathrm{n}. \tag{1.16}$$

The analysis of pion-nucleon and pion-deuteron reactions led to the conclusion that the pion spin is zero. The pions are bosons and obey *Bose statistics*, required for the treatment of particles with integer spin.

1.4 Antiparticles

For each particle in nature there is a corresponding antiparticle, with the same mass, and with charge of the same magnitude and opposite sign. This concept was established around 1930 with the development of relativistic quantum mechanics by Dirac and had its first experimental confirmation with the discovery of the positron (*antielectron*) by Anderson [An32] in 1932. The proton antiparticle (*antiproton*) was detected by Chamberlain and collaborators in 1955 [Ch55], using the 6 GeV bevatron at the University of California.

The first studies of the reaction $\bar{\mathrm{p}} + \mathrm{p}$, where $\bar{\mathrm{p}}$ represents the antiproton, have shown that in the great majority of cases this reaction leads to the annihilation of the $\mathrm{p}\bar{\mathrm{p}}$ pair with production of pions, but in 0.3% of cases it is able to form the $\mathrm{n}\bar{\mathrm{n}}$ pair, where $\bar{\mathrm{n}}$ is the antiparticle of the neutron, or *antineutron*. It was in this way that, in 1956, Cork and collaborators [Co56] first detected the antineutron, using antiprotons emerging from a beryllium target bombarded with 6.2 GeV protons.

Antiprotons and antineutrons are *antinucleons*. The magnitude of every quantity associated to some particle is identical to that of the corresponding antiparticle, but, as we shall see soon, there are, besides charge, other quantities for which the values for particles and antiparticles have opposite signs.

The mesons π^+ and π^- are antiparticles of each other. In this case it is not important to define which is the particle and which is the antiparticle, since mesons are not normal constituents of matter. In the case of π^0, particle and antiparticle coincide, since charge and magnetic moment are zero.

1.5 Inversion and Parity

Apart from rotational invariance (discussed in Appendix A), a system can have another important spatial symmetry, namely, that with respect to the inversion of coordinates. Let the quantum state of a particle be described by the wavefunction $\Psi(\mathbf{r})$. The *parity* of this state is connected to the properties of the wavefunction by an inversion of coordinates

$$\mathbf{r} \rightarrow -\mathbf{r}. \tag{1.17}$$

If

$$\Psi(-\mathbf{r}) = +\Psi(\mathbf{r}) \tag{1.18}$$

we say that the state has *positive parity*, and if

$$\Psi(-\mathbf{r}) = -\Psi(\mathbf{r}) \tag{1.19}$$

we say that the state has *negative parity*. An inversion of coordinates about the origin is represented in quantum mechanics by the operator Π, where

$$\Pi\Psi(\mathbf{r}) = \Psi(-\mathbf{r}). \tag{1.20}$$

Π is called the *parity operator*. The eigenvalues of Π are ± 1 (since $\Pi^2 = 1$):

$$\Pi\Psi(\mathbf{r}) = \pm\Psi(\mathbf{r}). \tag{1.21}$$

If the potential that acts on the set of particles is an even function, that is, $V(\mathbf{r}) = V(-\mathbf{r})$, the parity operator commutes with the Hamiltonian and the parity remains constant in time, that is, it is *conserved*.

From the analysis of a system of two particles 1 and 2 that do not interact, described by the product wavefunction

$$\Psi(\mathbf{r}_1)\,\Psi(\mathbf{r}_2), \tag{1.22}$$

the parity of the system is the product of the parities of each particle. That is, the parity is a multiplicative quantum number.

Besides the parity connected to its spatial state, a particle can also have an intrinsic parity. In this case the total parity is the product of the intrinsic and spatial parities. In processes where no particle is created or destroyed, the intrinsic parities of the particles are irrelevant. In reactions where particles are created and destroyed, the application of parity conservation must include the intrinsic parities of the particles.

Since $\Pi = \Pi^{-1}$, there is no distinction between the active and passive viewpoints (see Appendix A). In Cartesian coordinates the inversion transformation means $(x, y, z) \rightarrow (-x, -y, -z)$, whereas in spherical polar coordinates

$$(r, \theta, \varphi) \rightarrow (r, \pi - \theta, \varphi + \pi). \tag{1.23}$$

Therefore $\sin\theta$ does not change, $\cos\theta$ changes sign, and the function $e^{-im\varphi}$ acquires the factor $(-)^m$.

Rotations commute with inversion, as can be easily understood from the geometrical picture and checked formally. This implies that if a state belonging to a rotational multiplet has a certain parity, this quantum number should be the same for all members of the multiplet. For the spherical function Y_{ll} given by equations (A.7) and (A.89) of Appendix A, we find parity $(-)^l$. Therefore we conclude that

$$\Pi Y_{lm}(\mathbf{n}) = Y_{lm}(-\mathbf{n}) = (-)^l Y_{lm}(\mathbf{n}). \tag{1.24}$$

If the particle has a positive intrinsic parity, the total parity will be $(-1)^l$. If the particle has a negative intrinsic parity, the total parity will be $(-1)^{l+1}$. The same result is clear for P_l which are polynomials of order l in $\cos\theta$. In particular, for the backward direction

$$P_l(-1) = (-)^l. \tag{1.25}$$

The operators, such as **r** or **p**, acting on a state with a certain parity, change this value to the opposite one. They can be called Π-odd operators.

For a particle in a spherically symmetric field, stationary wavefunctions have a certain value of the orbital momentum l,

$$\psi(\mathbf{r}) = R_l(r) Y_{lm}(\mathbf{n}), \tag{1.26}$$

where $R_l(r)$ is a radial function. We see that parity for single-particle motion is uniquely determined by the orbital momentum. This is not the case in many-body systems, where total momentum and parity are independent in general.

From an analysis of pion and nucleon reactions, one concludes that the intrinsic parity of the former is $\Pi_\pi = -1$ and for the nucleons $\Pi_n = \Pi_p = +1$.

1.6 Isospin and Baryonic Number

The elementary particles exist in groups of approximately the same mass, but with different charges. The mass of the neutron, for example, is about the same as that of the proton, and the mass of the neutral pion, π^0, is approximately equal to that of the charged pions, π^+ and π^-. In 1932, Heisenberg [He32] suggested that the proton and the neutron could be seen as two *charge states* of a single particle, using the name nucleon to identify this particle.

In the theory of atomic spectra, a state that has *multiplicity* $2s+1$ has spin s[1]. It is common, however, to refer to the *spin quantum number* s simply as *spin* s (for more details on multiplets, see Appendix A). This is also true for the orbital and total angular momentum. One example is the Zeeman effect, which is the energy splitting among the $2s+1$ states of an atom in a magnetic field. The spin s is identified with the angular momentum of the system, and operators for the components of this angular momentum, s_x, s_y, s_z, can be defined. The quantum commutation rules are defined as

$$[s_x, s_y] = i\hbar s_z, \quad [s_y, s_z] = i\hbar s_x, \quad [s_z, s_x] = i\hbar s_y. \tag{1.27}$$

The nucleon has, relative to its charge, a multiplicity $2 \times \frac{1}{2} + 1 = 2$. By analogy with the theory of atomic spectra, we create a quantity called *isospin*, $t = \frac{1}{2}$, to obtain the multiplicity $2t + 1 = 2$. Isospin cannot be identified with an angular momentum, and does not have any connection with the spatial properties of the nucleon. Nevertheless, we can introduce an isospin space, or charge space, where the isospin can be treated as a set of three components t_x, t_y, t_z, satisfying the same commutation rules as the spin

$$[t_x, t_y] = i\hbar t_z, \quad [t_y, t_z] = i\hbar t_x, \quad [t_z, t_x] = i\hbar t_y. \tag{1.28}$$

Thus we can deal with the isospin in the same way we deal with the angular momentum. Since the square of the spin, $\mathbf{s}^2$, has eigenvalues $s(s+1)$, the square of the isospin, $\mathbf{t}^2$, has eigenvalues $t(t+1)$. Addition of the isospins of several particles can be treated with the

[1] The spin is a vector with modulus $\hbar\sqrt{s(s+1)}$.

vector model for addition, used in atomic spectra theory. For example, when we add two spins $\frac{1}{2}$, the total spin can be 0 or 1. This is also the case of the sum of isospins of two nucleons (each one has spin $\frac{1}{2}$).

The $2s+1$ states of a system with spin s are denoted by the $2s+1$ distinct values of the z component of $\mathbf{s}$ (in units of $\hbar$):

$$s_z = -s,\ -s+1,\ \ldots,\ s-1,\ s. \tag{1.29}$$

By analogy, the $2t+1$ states of a system with isospin t are denoted by the $2t+1$ distinct values of the t_z component,

$$t_z = -t,\ -t+1,\ \ldots,\ t-1,\ t. \tag{1.30}$$

The direction of the third axis in charge space is chosen in such a way that $t_z = +\frac{1}{2}$ for the proton and $t_z = -\frac{1}{2}$ for the neutron.

The nucleons can be represented by a "two-level" system with two basis states

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1.31}$$

The states (1.31) have a certain electric charge $z_p = 1$ and $z_n = 0$ in units of e, that is, they are the eigenstates of the charge operator Q. Referring to the basis (1.31), in analogy to the usual spin (A.86), as z-representation, we can say that this quantization axis is related to the interaction with the electromagnetic field that allows one to distinguish between the two charge states of the nucleon.

Let us call the spinor space represented by the basis vectors (1.31) *charge space*. All operators in this space are 2×2 matrices as in spinor states of a spin $\frac{1}{2}$ particle. We can construct the full set of matrices acting in this two-dimensional space of the unit matrix and matrices $\tau_{1,2,3}$ defined exactly as the Pauli matrices (A.82) in spin space. Evidently, the charge operator is

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(1 + \tau_z). \tag{1.32}$$

We can also introduce the off-diagonal operators inducing transitions between the charge states of the nucleon. The operator raising the charge is

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau_+ p = 0, \quad \tau_+ n = p; \tag{1.33}$$

the lowering operator is

$$\tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = (\tau_+)^\dagger, \quad \tau_- p = n, \quad \tau_- n = 0. \tag{1.34}$$

The operators $\tau_\pm$ are built of the Pauli matrices

$$\tau_\pm = \tau_x \pm i\tau_y, \tag{1.35}$$

in the same way as the raising and lowering operators $J_\pm = J_x \pm iJ_y$ of any angular momentum operator, as explained in Appendix A. We can combine the matrices $\tau_{x,y,z}$ into a matrix "vector" $\boldsymbol{\tau}$ that is completely analogous to the vector $\boldsymbol{\sigma}$ of spin Pauli matrices.

Continuing this analogy, we speak about the *isospin* of the nucleon

$$\mathbf{t} = \frac{1}{2}\boldsymbol{\tau}, \tag{1.36}$$

which acts in 2 × 2 "isospace" with the basis (1.31), where the basis states have a certain charge

$$Z = \frac{1}{2} + t_z. \tag{1.37}$$

The correct full spelling of the "isospin" is "isobaric" spin, which unifies the *isobars*, these being states with the same mass number, as in, for instance, the proton and the neutron or nuclei with the same sum $A = Z + N$ of the proton and neutron numbers that coincides with the total baryonic charge B of a nucleus, but not "isotopic," which would relate the *isotopes* having the same electric charge Z at different masses A. In isospin language, the proton and neutron are the states with different projection of the isospin onto the z-axis of isospace,

$$t_3 p = \frac{1}{2}p, \quad t_3 n = -\frac{1}{2}n. \tag{1.38}$$

Here we use the convention accepted in particle physics. In nuclear physics the isospin projections are traditionally assigned in the opposite way, $-\frac{1}{2}$ for the proton, and $+\frac{1}{2}$ for the neutron. Then for stable nuclei, having as a rule more neutrons than protons, the total projection of the isospin would be positive; the charge operator (1.37) is redefined correspondingly.

The pion has three charge states and thus has isospin $t = 1$; the three pions form a charge *multiplet*, or isospin multiplet, with multiplicity $2t + 1 = 3$. The state $t_z = +1$ is attributed to the π^+, $t_z = 0$ to the π^0, and $t_z = -1$ to the π^-. This is connected to the convention that was adopted for the nucleons and is necessary for the validity of (1.39) below.

The isospin magnitude is an invariant quantity in a system governed by the strong interaction. In electromagnetic interactions this quantity is not necessarily conserved, and we shall verify below that this is the only conservation law that has different behavior in relation to these two forces.

The number of nucleons before and after a reaction is always the same (see, for example, eqs. (1.13) and (1.14)–(1.16). This suggests the introduction of a new quantity, B, called *baryonic number*, that is always conserved in reactions. We attribute to the proton and to the neutron the baryonic number $B = 1$, and to the antiproton and antineutron $B = -1$. To the pions we ascribe $B = 0$ (also for electrons, neutrinos, muons, and photons). In this way the conservation of baryonic number is extended to all reactions. This principle is extended to the leptons, defining a *leptonic number*, which is also conserved in reactions.

From the isospin and baryonic number definition we can write the charge q, in units of e, as

$$Z = t_z + \frac{B}{2}. \tag{1.39}$$

Since the antiparticle of a particle of charge q and baryonic number B has charge $-q$ and baryonic number $-B$, it must have also a third isospin component $-t_z$, where t_z is the isospin z-component of the corresponding particle.

1.7 Isospin Invariance

The *mirror symmetry* of strong interaction implies invariance under the charge symmetry transformation $p \leftrightarrow n$. By virtue of this symmetry the proton and neutron states are degenerate. Since their electromagnetic properties are different, this is equivalent to the statement that their mass difference comes exclusively from electromagnetic interactions, supposedly on the quark level. The charge symmetry transformation is a particular case of SU(2) transformations (see Appendix A). Moreover, *isospin invariance* assumes that the strong Hamiltonian is invariant under all elements of the isospin group. In this case we have full rotational invariance in isospin space, and the stationary states can be labeled by the conserved quantum number total isospin, T:

$$\mathbf{T} = \sum_a \mathbf{t}_a, \tag{1.40}$$

which is the analog of the total angular momentum in isospace related to the eigenvalues of the isospin "length," $\mathbf{T}^2 = T(T+1)$. $\mathbf{t}_a$ represents the isospin of nucleon a.

Since the algebraic properties of spin and isospin are identical, the allowed values of T are quantized to be integer (half-integer) in a system of an even (odd) number of nucleons. They give rise to degenerate *isomultiplets* with given T that contain $2T+1$ states with projections $T_z = -T, \dots, +T$ or, equivalently, with the charge [see (1.37)]

$$Z = \sum_a \left(\frac{1}{2} + t_{za}\right) = \frac{A}{2} + T_z, \tag{1.41}$$

where t_{za} is the z-component of $\mathbf{t}_a$.

The isospin invariance of the strong interaction Hamiltonian H_s can be written as the conservation law

$$[\mathbf{T}, H_s] = 0. \tag{1.42}$$

In the case of stationary states, all $2T+1$ states of a multiplet would have the same energy in the limit of exact isospin invariance. Let us emphasize that the states within a given isomultiplet belong to different nuclei (the same A but different Z). They are frequently called *isobaric analog* states (IAS). The conservation law (1.42) is certainly exact for the component T_z related to the electric charge, eq. (1.41). If we forget for a moment about electromagnetic interactions that single out the z-axis and violate the isotropy of isospace, we can classify all nuclear states by isomultiplets. All states in a given nucleus (the "vertical" scale) have the same

$$T_z = \frac{1}{2}(Z - N) = Z - \frac{A}{2}. \tag{1.43}$$

They belong to various isomultiplets (the "horizontal" scale). The allowed values of the magnitude T of the isospin of the nucleus (Z, A) cannot be less than the value of projection T_z, (1.43), but they cannot exceed the maximum value $A/2$,

$$\frac{1}{2}(Z - N) \leq T \leq \frac{1}{2}(Z + N). \tag{1.44}$$

We have already mentioned that the introduction of isospin does not increase the number of nuclear degrees of freedom, or the number of possible states. This is just a convenient classification associated with the invariance (1.42) of strong interactions. This classification is actually related to the permutational symmetry of the many-body wavefunction in the "normal" coordinate and spin variables. If the effects violating isospin invariance, in particular, due to electromagnetic interactions, can be neglected, we have an approximate isospin symmetry.

The concept of isospin SU(2) invariance is generalized to *higher symmetries* in quantum chromodynamics (see below), where it is related to the fact that the two lightest quarks, u (up) and d (down), have similar masses and interactions. In the approximation that neglects the difference of the u, d quarks from the s (strange) quark, the carrier of the strangeness, we have already three fundamental objects so that the corresponding invariance group is SU(3), or SU(6) if the interactions do not depend on spins.

1.8 Magnetic Moment of the Nucleons

A charged particle rotating around an axis can be visualized as a system equivalent to a small ring carrying an electric current. To this current is associated a magnetic dipole moment $\boldsymbol{\mu}$ that is related to the particle angular momentum $\mathbf{L}$ through $\boldsymbol{\mu} = e\mathbf{L}/2mc$, where e is the charge and m the mass of the particle. It is common to write

$$\boldsymbol{\mu}_L = \frac{eg_L}{2mc}\mathbf{L}, \tag{1.45}$$

where the factor g_L is introduced. It is called *orbital g factor*, equal to 1 for protons and 0 for neutrons.

However, as we have already discussed, a particle can have an intrinsic angular momentum $\mathbf{s}$. Thus it is fair to admit that an intrinsic magnetic moment can also be associated to a particle, given by

$$\boldsymbol{\mu}_S = \frac{eg_S}{2mc}\mathbf{s}, \tag{1.46}$$

where the constant g_S, the *spin g-factor*, does not necessarily have the same value g_L adequate to classical variables, since $\mathbf{s}$ and $\boldsymbol{\mu}_S$ have pure quantum origin. In fact, from a relativistic treatment of quantum mechanics using the Dirac equation, a value $g_S = 2$ for spin $\frac{1}{2}$ charged particles is obtained [Di30]

The universal constant (using m as the proton mass)

$$\mu_N = \frac{e\hbar}{2mc} = 5.05 \times 10^{-27}\ \frac{\text{Joule m}^2}{\text{Weber}} \tag{1.47}$$

is known as the *nuclear magneton*, by analogy with the Bohr magneton defined for the electron. The nuclear magneton is used as a unit for magnetic moments and it is convenient to note that, owing to the proton mass in the denominator, its value is about 1800 times less than its electronic equivalent. With it, (1.45) and (1.46) can be rewritten as

$$\boldsymbol{\mu}_L = \mu_N g_L \frac{\mathbf{L}}{\hbar} \quad \text{and} \quad \boldsymbol{\mu}_S = \mu_N g_S \frac{\mathbf{s}}{\hbar}. \tag{1.48}$$

The prediction $g_S = 2$ obtained by Dirac works very well for the electron. For the proton and the neutron the values found experimentally are

$$g_S = 5.5856 \ \text{(proton)} \quad \text{and} \quad g_S = -3.8262 \ \text{(neutron)}, \tag{1.49}$$

when the expected values from Dirac's theory would be 2 and 0. The discrepancy above can be explained in part by the *virtual dissociation* of the nucleons. As we have already established, the uncertainty principle allows a nucleon to emit and reabsorb a pion during a time interval $\Delta t \sim \hbar/m_\pi c^2$, as described by the diagrams of figure 1.1b,c. One proton can be dissociated into a neutron and a π^+. The π^+ has spin zero and does not have intrinsic magnetic moment, but it can contribute to the proton magnetic moment due to its orbiting around the neutron. Supposing that this process conserves angular momentum and parity, one can show that the π^+ orbiting is in the same direction as the initial intrinsic spin of the proton. The effect of the virtual production of a π^+ is therefore to increase the proton magnetic moment, as is experimentally observed.

In the same way the π^- contributes to the neutron magnetic moment. There is a small contribution to the magnetic moment due to the proton orbit and spin but the largest contribution comes from the π^- orbiting, due to its small mass; the orbital magnetic moment depends on the inverse of the mass of the particle in orbit (1.47), and $m_\pi \ll m_\text{p}$. The contribution is negative due to the charge of the π^- and because it orbits in the same direction as the neutron spin. So, we expect that the neutron intrinsic magnetic moment is less than zero, as is experimentally verified. This analysis is only qualitative. A more exact explanation of the magnetic moments of the nucleons is still the object of theoretical studies.

The antiproton, being a particle with negative charge, has magnetic dipole moment in a direction opposite to that of the angular momentum and thus $g_L = -1$. The values given by (1.49) also have opposite sign for the antiparticles. Then, the antiproton magnetic moment points contrary to the spin and that of the antineutron stays aligned to the spin.

1.9 Strangeness and Hypercharge

In 1947, particles with properties different from those known at that time were found in cosmic rays. They were afterward (1953) observed in the laboratory. In cloud chambers, where their trajectories were detected and photographed, they appeared as a pair of tracks in form of a V, making clear that two particles were created simultaneously. These *strange particles* or *V-particles*, as they were initially known, form two distinct groups. One of them consists of particles heavier than the nucleons and decaying into them, called *hyperons*. The symbols Λ, Σ, Ξ, and Ω are utilized for the several hyperons. Because they decay into nucleons, hyperons are baryons and have baryonic number 1. They also have spin $\frac{1}{2}$, and they are fermions. The other group of strange particles are bosons with spin 0 and are called *K-mesons*, or *kaons*.

Typical reactions involving strange particles are

$$
\begin{aligned}
&\Lambda^0 \to p + \pi^-, \qquad && \Lambda^0 \to n + \pi^0,\\
&\Xi^- \to \Lambda^0 + \pi^-, && \Xi^0 \to \Lambda^0 + \pi^0,\\
&K^0 \to \pi^+ + \pi^-, && \Lambda^0 \to \pi^- + p,\\
&\pi^- + p \to K^0 + \Lambda^0. &&
\end{aligned}
\tag{1.50}
$$

The *interaction time* for reactions involving nucleons and pions is obtained approximately by the time in which a pion, with velocity near that of light, travels a distance equal to the nuclear force range. This time is about $r/c \simeq 10^{-23}$ s, which is much less than the mean lifetime of the Λ^0 ($\tau = 2.5 \times 10^{-10}$ s), or of other strange particles. On the other side, it was found experimentally that the rate at which or other strange particles are produced is consistent with an interaction time on the order of 10^{-23} s. To explain why strange particles were produced so fast but decayed so slowly, Pais [Pa52] suggested that strong interactions (those acting between nucleons, or between pions and nucleons) are responsible for the production of strange particles. The reactions in which only one strange particle takes part, as in its decay, would proceed through weak interactions, similar to β-decay or the decay of muons or charged pions.

In 1953, Gell-Mann [Ge53] and Nishijima and Nakano [NN53] showed that the production of strange particles could be explained by the introduction of a new quantum number, *strangeness*, and postulating that strangeness is conserved in strong interactions. For example, two strange particles, but with opposite strangeness, could be produced by means of the strong interaction in a collision between a pion and a nucleon. Strangeness, however, is not conserved in the decay of a strange particle, and this decay is attributed to the weak interaction.

The great number of hadrons, as shown in table 1.1, and their apparently complex distribution led several investigators to question if these particles might be complex structures composed by the union of simpler entities. Models were proposed for those structures and, after some unsuccessful attempts, a model independently created by M. Gell-Mann [Ge64] and G. Zweig [Zw64], in 1964 imposed itself and has gained credibility over time. The inspiration for that model came from the symmetries observed when one put mesons and baryons in plots of strangeness versus the t_z-component of isospin, as shown in figure 1.2. The type of observed symmetry is a characteristic of the group called SU(3), where three basic elements can generate singlets (the mesons η' and ϕ), octets (the other eight mesons of the figures above and the eight spin $\frac{1}{2}$ baryons) and decuplets (the spin $\frac{3}{2}$ baryons). These three basic elements, initially conceived only as mathematical entities able to generate the necessary symmetries, ended up acquiring the status of real elementary particles, for which Gell-Mann coined the name *quarks*. To obtain hadronic properties, these three quarks, presented in the *flavors up, down*, and *strange*, must have the characteristic values shown in table 2.

Table 1.1 Attributes of particles that interact strongly.

n	B	S	t	t_z	s	m (MeV/c^2)
p	+1	0	1/2	+1/2	1/2	938.272
n	+1	0	1/2	−1/2	1/2	939.565
$\bar{p}$	−1	0	1/2	−1/2	1/2	938.272
$\bar{n}$	−1	0	1/2	+1/2	1/2	939.565
Λ	+1	−1	0	0	1/2	1115.68
Σ^+	+1	−1	1	+1	1/2	1189.4
Σ^0	+1	−1	1	0	1/2	1192.6
Σ^-	+1	−1	1	−1	1/2	1197.4
$\bar{\Lambda}$	−1	+1	0	0	1/2	1115.68
$\bar{\Sigma}^+$	−1	+1	1	−1	1/2	1189.4
$\bar{\Sigma}^0$	−1	+1	1	0	1/2	1192.6
$\bar{\Sigma}^-$	−1	+1	1	+1	1/2	1197.4
Ξ^0	+1	−2	1/2	+1/2	1/2	1315
Ξ^-	+1	−2	1/2	−1/2	1/2	1321
$\bar{\Xi}^0$	−1	+2	1/2	−1/2	1/2	1315
$\bar{\Xi}^-$	−1	+2	1/2	+1/2	1/2	1321
Ω^-	+1	−3	0	0	3/2	1672
π^0	0	0	1	0	0	134.976
π^+	0	0	1	+1	0	139.567
π^-	0	0	1	−1	0	139.567
K^+	0	+1	1/2	+1/2	0	493.7
K^-	0	−1	1/2	−1/2	0	493.7
K^0	0	+1	1/2	−1/2	0	497.7
$\bar{K}^0$	0	−1	1/2	+1/2	0	497.7

Notes: The baryons are the particles with baryonic number $B \neq 0$; the mesons have $B = 0$. S is strangeness, t the isotopic spin and t_z its projection; s is the particle spin and m its mass. Baryons have positive intrinsic parity; mesons have negative ones.

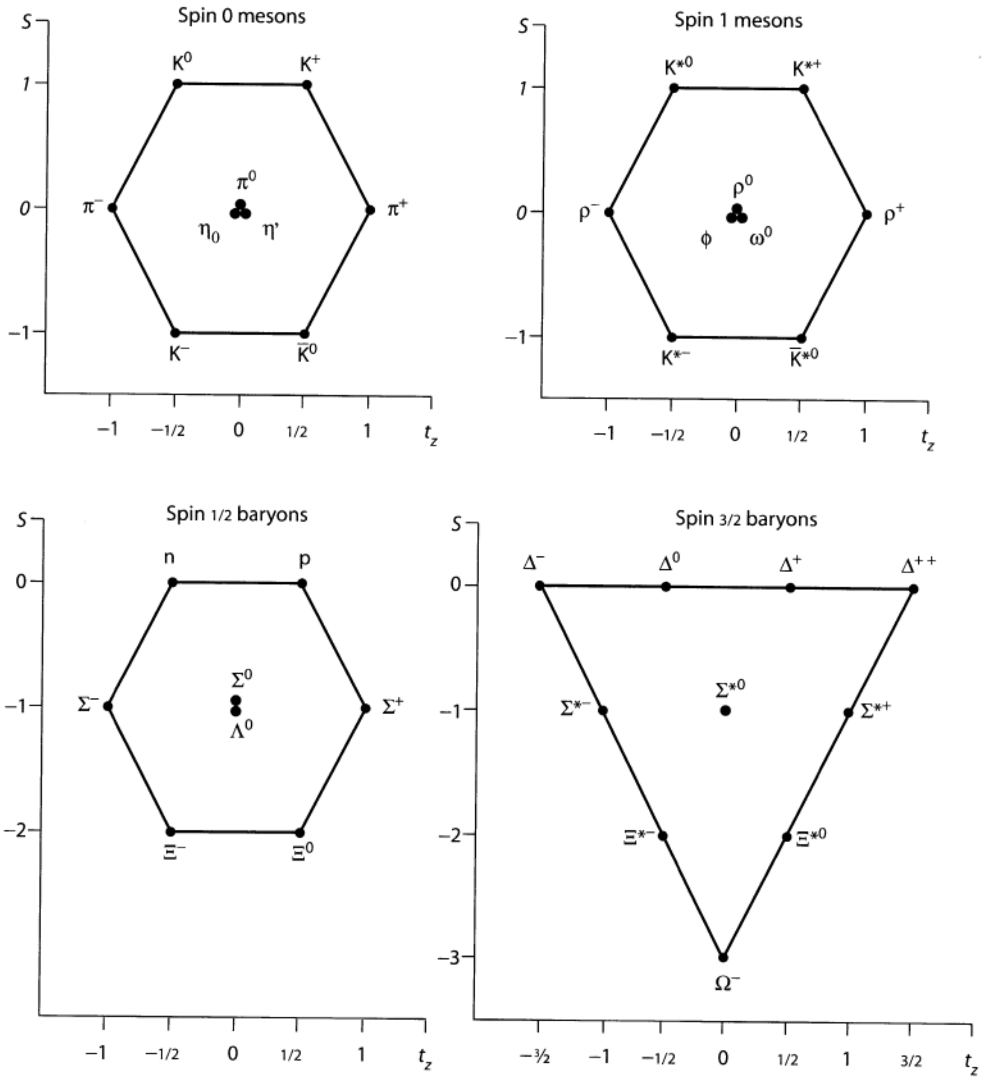

Figure 1.2 Strangeness versus t_z-component of isospin for the several types of hadrons.

The most striking fact is that, for the first time, the existence of particles with fractional charge (a fraction of the electron charge) is admitted. We can in this way construct a nucleon by composing three quarks (neutron = udd), and it is natural to attribute to quarks a baryonic number $B = \frac{1}{3}$. The pions, in turn, are obtained by the conjunction of a quark and an antiquark, ($\pi^+ = u\bar{d}$), ($\pi^0 = d\bar{d}$), ($\pi^- = d\bar{u}$), where the properties of the antiparticle for the quarks are obtained in the conventional way.

To reproduce the other baryons and mesons, the strange quarks have to play a role, and a hyperon like Σ^0, for example, has the constitution ($\Sigma^0 = $ uds), while a meson has the constitution ($K^+ = u\bar{s}$). It is convenient to say at this point that a certain combination of quarks does not necessarily lead to only one particle. In the case of the combination above, we also have the possibility to build the hyperon ($\Sigma^{*0} = $ uds). The reason for this is

Table 1.2 Quark characteristic quantum numbers.

Flavor	Charge	Spin	Strangeness
up	+2/3	1/2	0
down	−1/3	1/2	0
strange	−1/3	1/2	−1

that, besides other quantum numbers that will be discussed later, a combination of three fermions can give rise to particles with different spin. If we consider as zero the quarks' total orbital angular momentum, which is true for all particles we discussed, the total spin of the three quarks can be $\frac{1}{2}$ or $\frac{3}{2}$. The hyperon Σ^0 corresponds to the first case and the hyperon Σ^{*0} to the second.

A first difficulty in the theory appears when we examine the particles ($\Delta^{++} = \text{uuu}$), ($\Delta^{-} = \text{ddd}$), ($\Omega^{-} = \text{sss}$). Since the three quarks in each case are fermions with $l = 0$, it is clear that at least two of them would be in the same quantum state, which violates the Pauli principle. To overcome this difficulty, a new quantum number was introduced, *color*: the quarks, besides the flavors up, down, or strange, would have a color, red (R), green (G), or blue (B), or *anticolor*, $\bar{\text{R}}$, $\bar{\text{G}}$, or $\bar{\text{B}}$. It is clear that, in the same way as flavor, color has nothing to do with the usual notion we have of that property. The introduction of this new quantum number solves the above difficulty; since now a baryon like Δ^{++} is written $\Delta^{++} = \text{u}_R\text{u}_G\text{u}_B$, the problems with the Pauli principle are eliminated. The addition of three new quantum numbers increases enormously the possibility of construction of hadrons, but a new rule comes to play, limiting the possible of color combinations: *all the possible states of hadrons are colorless,* where colorless in this context means absence of color or white color. White is obtained when, in a baryon, one adds three quarks, one of each color. In this sense the analogy with the common colors works, since the addition of red, green, and blue gives white. In a meson, absence of color results from the combination of a color and the respective anticolor. Another way to present this property is to understand the anticolor as the complementary color. In this case, the analogy with the common colors also works and the pair color-anticolor also results in white.

The concept of color is not only useful to solve the problem with the Pauli principle. It has a fundamental role in quark interaction processes. The accepted theory for this interaction establishes that the force between quarks works by the exchange of massless particles, with spin 1, called *gluons.* These gluons always carry one color and one different anticolor, and in the mediation process they interchange the respective colors; one example is seen in figure 1.3. One can also see in figure 1.4b that the gluons themselves can emit gluons.

The fields around hadrons where exchange forces act by means of colors are denominated *color fields,* and the gluons, the exchanged particles, turn out to be the field particles of the strong interaction. In this task they replace the pions that, in the new scheme, are composite particles. The fact that the gluons have colors and can interact mutually makes the study of color fields (quantum chromodynamics) particularly complex.

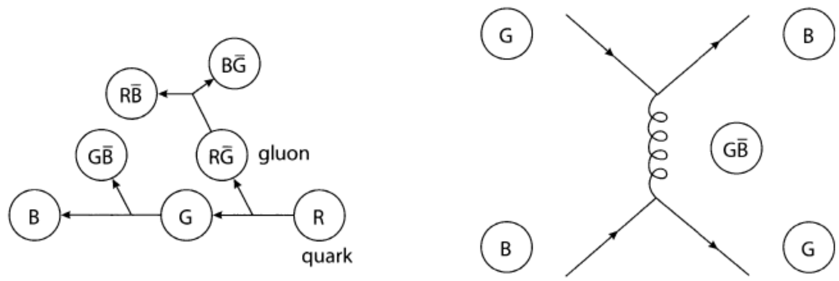

Figure 1.3 (a) Forces between quarks mediated by gluon exchange. (b) Diagram showing how a quark B changes to a quark G, and vice-versa, by the exchange of a gluon $G\bar{B}$.

Despite the success of the *quark model*, new difficulties arose and, in 1970, with the purpose of explaining some decay times in disagreement with the predictions of the model, S. L. Glashow, J. Iliopoulos, and L. Maiani [Gl70] proposed the existence of a fourth quark whose flavor has received the designation *charm* (c). This c quark has a charge of $+\frac{2}{3}$; it has strangeness zero but has a new quantum number, the charm C, with an attributed value $C = 1$. The prediction of quark c received experimental confirmation in 1974, when two independent laboratories detected a new particle, called Ψ by the Stanford linear accelerator group (SLAC) and J by the Brookhaven National Laboratory team. The particle J/Ψ, as it is commonly designated, is interpreted as a $c\bar{c}$ state, called *charmonium*, by analogy with positronium $e\bar{e}$. The existence of particles with charm introduces some complications in the symmetries of figure 1.2: an axis with the new quantum number is added and the new symmetries have to be sought in three-dimensional space.

In 1977, a group of resonances in 10 GeV proton-proton reactions pointed to the existence of a new meson that received the name Υ and that led to the proposition of a new quark. This quark, b (from *bottom* or *beauty*), has charge $-\frac{1}{3}$ and a new quantum number, *beauty*, B^*. The quark b has $B^* = -1$.

Theoretical reasons imply that quarks exist in pairs, and this led to a sixth flavor, which corresponds to quark t (from *top* or *true*), with charge $+\frac{2}{3}$. This quark was identified in experiments conducted at Fermilab in 1993 [Ab94].

The theory of quarks, with its colors and flavors, has created a scheme in which a great number of experimental facts can be explained. High energy electron beams have indeed detected an internal structure in nucleons with all the features of quarks [Fr91]. However, one can never pull out a quark from a hadron and study its properties separately. To eliminate this possibility, a theory of *asymptotic freedom* was developed *confining* quarks permanently to the hadrons. One consequence is that their mass cannot be directly determined, since it depends on the binding energies, which are also unknown. The quark model enjoys high prestige in the theory of elementary particles, and there is a substantial reduction in the number of elementary particles, that is, point particles without an internal structure. These are the quarks, the leptons, and the field bosons. An outline of the properties of these particles is shown in table 1.3.

Table 1.3 Properties of the elementary particles.

Quarks	Charge	Spin	Strangeness	Charm	Beauty	Truth
u	+2/3	1/2	0	0	0	0
d	−1/3	1/2	0	0	0	0
s	−1/3	1/2	−1	0	0	0
c	+2/3	1/2	0	1	0	0
b	−1/3	1/2	0	0	−1	0
t	+2/3	1/2	0	0	0	1

Leptons	Mass (MeV/c^2)	Charge	Spin	Half-life (s)
e^-	0.511	−1	1/2	∞
ν_e	0	0	1/2	∞
μ^-	105.66	−1	1/2	2.2×10^{-6}
ν_μ	0	0	1/2	∞
τ^-	1784	−1	1/2	3.4×10^{-13}
ν_τ	0	0	1/2	∞

Field particles	Mass (GeV/c^2)	Charge	Spin
Photon	0	0	1
$W^\pm$	81	1	1
Z^0	93	0	1
Gluons	0	0	1
Graviton	0	0	2

Notes: In the upper table each quark can appear in three colors, R, G, and B. Only one member of the particle-antiparticle pair appears in the table

1.10 Quantum Chromodynamics

It is well known that *quantum chromodynamics* (QCD) is the fundamental theory for strongly interacting particles. In this section we give a brief description of QCD. The formalism is best described in terms of the Lagrangian formalism. Within this approach, the *Euler-Lagrangian equations* yield the equations of motion for the fundamental particles. If the reader is not familiar with relativistic quantum mechanics and the notation used in this section, it is recommended to read Appendix D.

Strong interaction is indeed the strongest force of nature. It is responsible for over 80% of the baryon masses, and thus for most of the mass of everything on Earth. Strong interactions bind nucleons in nuclei, which, being then dressed with electrons and bound into molecules by the much weaker electromagnetic force, give rise to the variety of the physical world.

Quantum chromodynamics is the theory of strong interactions. The fundamental degrees of freedom of QCD, quarks and gluons, are already well established even though they cannot be observed as free particles, but only in color neutral bound states (*confinement*). Today, QCD has firmly occupied its place as part of the *standard model of particle physics* (for a good introduction to the field, see [HM84]). However, understanding the physical world does not only mean understanding its fundamental constituents; it means mostly understanding how these constituents interact and bring into existence the entire variety of physical objects composing the universe. Here, we try to explain why high energy nuclear physics offers us unique tools to study QCD.

QCD emerges when the naïve quark model is combined with local SU(3) gauge invariance. We will just summarize some of the main accomplishments in QCD. For an introduction to the concepts discussed here, we refer to, e.g., [PS95]. One can define a quark-state "vector" with three components (in this section I use $\hbar = c = 1$),

$$q(x) = \begin{pmatrix} q^{\text{red}}(x) \\ q^{\text{green}}(x) \\ q^{\text{blue}}(x) \end{pmatrix}, \tag{1.51}$$

where $q^{\text{color}}(x)$ are field quantities that depend on the space-time coordinate $x = (t, \mathbf{r})$. The transition from quark model to QCD is made when one decides to treat color similarly to the electric charge in electrodynamics. The entire structure of electrodynamics emerges from the requirement of local gauge invariance, that is, invariance with respect to the phase rotation of the electron field, $\exp(i\alpha(x))$, where the phase α depends on the space-time coordinate. One can demand similar invariance for the quark fields, keeping in mind that while there is only one electric charge in quantum electrodynamics (QED), there are three color charges in QCD.

To implement this program, one requires the free quark Lagrangian,

$$\mathcal{L}_{\text{free}} = \sum_{q=u,d,s,\ldots} \sum_{\text{colors}} \bar{q}(x) \left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - m_q \right) q(x)\,, \tag{1.52}$$

to be invariant under rotations of the quark fields in color space,

$$U : q^j(x) \to U_{jk}(x) q^k(x), \tag{1.53}$$

with $j, k \in \{1, 2, 3\}$ (we always sum over repeated indices). Since the theory we build in this way is invariant with respect to these "gauge" transformations, all physically meaningful quantities must be gauge invariant. In (1.52), $\partial/\partial x_\mu = (\partial/\partial t,\ \nabla)$, and $\gamma_\mu = (\gamma_0, \gamma_i)$ are the 4×4 Dirac matrices, defined in Appendix D.

The indices (i, j, k) can only have values 1, 2, 3 (or, equivalently, x, y, z), and the index 0 means the time component of the matrix γ_μ . The γ-matrices are functions of the Pauli matrices σ (see Appendix D).

The contraction of two four-vectors is defined as $A^\mu B_\mu = A^0 B^0 - \mathbf{A} \cdot \mathbf{B}$. In (1.52), $\bar{q}(x)$ is a matrix multiplication of the complex conjugate of the transpose of (1.51) and the Dirac matrix γ_0, that is, $\bar{q}(x) = q^\dagger(x)\gamma_0$ (see Appendix D).

In electrodynamics, there is only one electric charge, and gauge transformation involves a single phase factor, $U = \exp(i\alpha(x))$. In QCD, one has three different colors, and U becomes a (complex-valued) unitary 3×3 matrix, that is, $U^\dagger U = UU^\dagger = 1$, with determinant Det $U = 1$. These matrices form the fundamental representation of the group SU(3), where 3 is the number of colors, $N_c = 3$. The matrix U has $N_c^2 - 1 = 8$ independent elements and can therefore be parameterized in terms of the 8 generators T^a_{kj}, $a \in \{1, \ldots, 8\}$ of the fundamental representation of SU(3),

$$U(x) = \exp\left(-i\phi_a(x)T^a\right). \tag{1.54}$$

By considering a transformation U that is infinitesimally close to the **1** element of the group, it is easy to prove that the matrices T^a must be Hermitian ($T^a = T^{a\dagger}$) and traceless (tr $T^a = 0$). The T^a's do not commute; instead, one defines the SU(3) structure constants f_{abc} by the commutator

$$\left[T^a, T^b\right] = i f_{abc} T^c. \tag{1.55}$$

These commutator terms have no analog in QED, which is based on the abelian gauge group U(1). QCD is based on a non-abelian gauge group SU(3) and is thus called a *non-abelian gauge theory*.

The generators T^a are normalized to

$$\mathrm{tr}\, T^a T^b = \frac{1}{2}\,\delta_{ab}, \tag{1.56}$$

where δ_{ab} is the Kronecker symbol. Useful information about the algebra of color matrices, and their explicit representations, can be found in many textbooks (see, e.g., [Fie89]).

Since U is x-dependent, the free quark Lagrangian (1.52) is not invariant under the transformation (1.53). In order to preserve gauge invariance, one has to introduce, following the familiar case of electrodynamics, the *gauge* (or "gluon") *field* $A^\mu_{kj}(x)$ and replace the derivative in (1.52) with the so-called *covariant derivative,*

$$\partial^\mu q^j(x) \to D^\mu_{kj} q^j(x) \equiv \left\{\delta_{kj}\partial^\mu - iA^\mu_{kj}(x)\right\} q^j(x), \tag{1.57}$$

where ∂^μ is the four-dimensional derivative $\partial^\mu = (\partial/\partial t,\ \nabla)$.

Note that the gauge field $A^\mu_{kj}(x) = A^\mu_a T^a_{kj}(x)$ as well as the covariant derivative are 3×3 matrices in color space. Note also that (1.57) differs from the definition often given in textbooks, because we have absorbed the strong coupling constant in the field A^μ. With the replacement given by (1.57), all changes to the Lagrangian under gauge transformations cancel, provided A^μ transforms as

$$U: A^\mu(x) \to U(x)A^\mu(x)U^\dagger(x) + iU(x)\partial^\mu U^\dagger(x). \tag{1.58}$$

(From now on, we will often not write the color indices explicitly.)

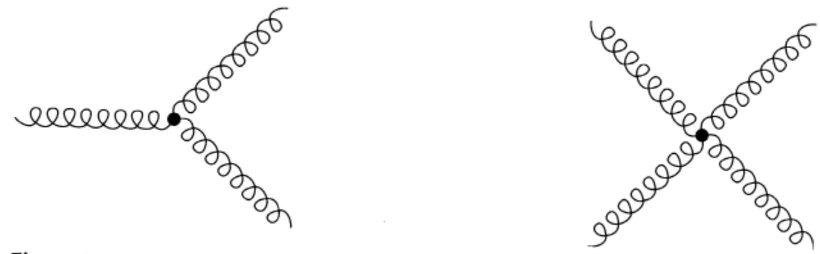

Figure 1.4 Due to the non-abelian nature of QCD, gluons carry color charge and can therefore interact with each other via these vertices.

The QCD Lagrangian then reads

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(x)\left(\mathrm{i}\gamma_\mu D^\mu - m_q\right) q(x) - \frac{1}{4}\mathrm{tr}\; G^{\mu\nu}(x) G_{\mu\nu}(x), \tag{1.59}$$

where the first term describes the dynamics of quarks and their couplings to gluons, while the second term describes the dynamics of the gluon field. The strong coupling constant g is the QCD analog of the elementary electric charge e in QED. The gluon field strength tensor is given by

$$G^{\mu\nu}(x) \equiv \mathrm{i}\left[D^\mu, D^\nu\right] = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x) - \mathrm{i}\left[A^\mu(x), A^\nu(x)\right]. \tag{1.60}$$

This can also be written in terms of the color components A_a^μ of the gauge field,

$$G_a^{\mu\nu}(x) = \partial^\mu A_a^\nu(x) - \partial^\nu A_a^\mu(x) + f_{abc} A_b^\mu(x) A_c^\nu(x). \tag{1.61}$$

For a more complete presentation, see modern textbooks like [Fie89], [ESW96], [Mu87].

The crucial difference, as will become clear soon, between electrodynamics and QCD is the presence of the commutator on the right-hand side of (1.60). This commutator gives rise to the gluon-gluon interactions shown in figure 1.4 that make the QCD field equations nonlinear: the color fields do not simply add like in electrodynamics. These nonlinearities give rise to rich and nontrivial dynamics of strong interactions.

Let us now turn to the discussion of the dynamical properties of QCD. To understand the dynamics of a field theory, one necessarily has to understand how the coupling constant behaves as a function of distance. This behavior, in turn, is determined by the response of the vacuum to the presence of external charge. The vacuum is the ground state of the theory; however, quantum mechanics tells us that the "vacuum" is far from empty—the uncertainty principle allows particle-antiparticle pairs to be present in the vacuum for a period of time inversely proportional to their energy. In QED, the electron-positron pairs have the effect of screening the electric charge; see figure 1.5. Thus, the electromagnetic coupling constant increases toward shorter distances. The dependence of the charge on distance (*running coupling constant*) is given by [HM84]

$$e^2(r) = \frac{e^2(r_0)}{1 + \dfrac{2e^2(r_0)}{3\pi} \ln \dfrac{r}{r_0}}, \tag{1.62}$$

which can be obtained by resuming (logarithmically divergent, and regularized at the distance r_0) electron-positron loops dressing the virtual photon propagator.

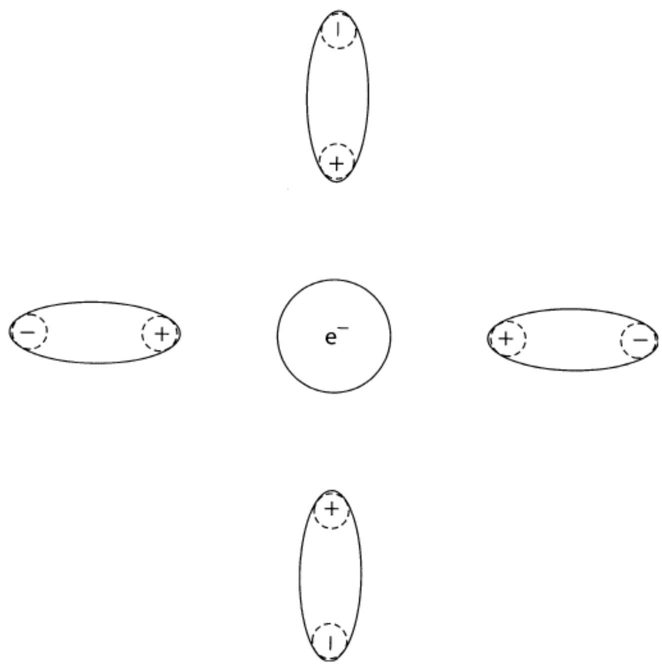

Figure 1.5 In QED, virtual electron-positron pairs from the vacuum screen the bare charge of the electron. The larger the distance, the more pairs are present to screen the bare charge and the electromagnetic coupling decreases. Conversely, the coupling is larger when probed at short distances.

The formula (1.62) has two surprising properties. First, at large distances r away from the charge which is localized at r_0, $r \gg r_0$, where one can neglect unity in the denominator, the "dressed" charge $e(r)$ becomes independent of the value of the "bare" charge $e(r_0)$—it does not matter what the value of the charge at short distances is. Second, in the local limit $r_0 \to 0$, if we require the bare charge $e(r_0)$ to be finite, the effective charge vanishes at any finite distance away from the bare charge! The screening of the charge in QED does not allow one to reconcile the presence of interactions with the local limit of the theory. This is a fundamental problem of QED, which shows that either i) it is not a truly fundamental theory, or ii) (1.62), based on perturbation theory, in the strong coupling regime gets replaced by some other expression with more acceptable behavior. The latter possibility is quite likely, since at short distances the electric charge becomes very large and its interactions with the electron-positron vacuum cannot be treated perturbatively.

Fortunately, because of the smallness of the physical coupling $\alpha_{em}(r) = e^2(r)/(4\pi) = 1/137$, this fundamental problem of the theory manifests itself only at very short distances $\sim \exp(-3/[8\alpha_{em}])$. Such short distances will probably always remain beyond the reach of experiment, and one can safely apply QED as a truly effective theory.

In QCD, as we are now going to discuss, the situation is qualitatively different, and corresponds to *anti*-screening—the charge is small at short distances and grows at larger distances. This property of the theory is called *asymptotic freedom* [GW73], [Po73].

While the derivation of the running coupling is conventionally performed by using field theoretical perturbation theory, it is instructive to see how these results can be illustrated by using the methods of condensed matter physics. Indeed, let us consider the vacuum as a continuous medium with a dielectric constant ϵ. The dielectric constant is linked to the magnetic permeability μ and the speed of light c by the relation

$$\epsilon\,\mu = \frac{1}{c^2} = 1. \tag{1.63}$$

Thus, a screening medium ($\epsilon > 1$) will be diamagnetic ($\mu < 1$), and conversely a paramagnetic medium ($\mu > 1$) will exhibit antiscreening, which leads to asymptotic freedom. In order to calculate the running coupling constant, one has to calculate the magnetic permeability of the vacuum. In QED one has [Fie89], [ESW96], [Mu87]

$$\epsilon_{\mathrm{QED}} = 1 + \frac{2e^2(r_0)}{3\pi}\ln\frac{r}{r_0} > 1. \tag{1.64}$$

So why is the QCD vacuum paramagnetic while the QED vacuum is diamagnetic? The energy density of a medium in the presence of an external magnetic field **B** is given by

$$u = -\frac{1}{2}4\pi\chi\mathbf{B}^2, \tag{1.65}$$

where the magnetic susceptibility χ is defined by the relation

$$\mu = 1 + 4\pi\chi. \tag{1.66}$$

When electrons move in an external magnetic field, two competing effects determine the sign of magnetic susceptibility:

- The electrons in the magnetic field move along quantized orbits, referred to as *Landau levels*. The current originating from this movement produces a magnetic field with opposite direction to the external field. This is the diamagnetic response, $\chi < 0$.
- The electron spins align along the direction of the external **B**-field, leading to a paramagnetic response ($\chi > 0$).

In QED, the diamagnetic effect is stronger, so the vacuum is screening the bare charges. In QCD, however, gluons carry color charge. Since they have a larger spin (spin 1) than quarks (or electrons), the paramagnetic effect dominates and the vacuum is antiscreening.

Based on the considerations given above, the energy density of the QCD vacuum in the presence of an external color-magnetic field can be calculated by using the standard formulas of quantum mechanics, see, for example, [LL65], by summing over Landau levels and taking account of the fact that gluons and quarks give contributions of different sign. Note that a summation over all Landau levels would lead to an infinite result for the energy density. In order to avoid this divergence, one has to introduce a cutoff Λ with dimension of mass. Only field modes with wavelength $\lambda \gtrsim 1/\Lambda$ are taken into account. The upper limit for λ is given by the radius of the largest Landau orbit, $r_0 \sim 1/\sqrt{gB}$, which is the only dimensional scale in the problem; the summation thus is made over the wave lengths satisfying

$$\frac{1}{\sqrt{|gB|}} \gtrsim \lambda \gtrsim \frac{1}{\Lambda}. \tag{1.67}$$

The result is [Nie81]

$$u_{\text{vac}}^{\text{QCD}} = -\frac{1}{2}B^2 \frac{11N_c - 2N_f}{48\pi^2} g^2 \ln \frac{\Lambda^2}{|gB|}, \tag{1.68}$$

where N_f is the number of quark flavors, and $N_c = 3$ is the number of colors. Comparing this with (1.65) and (1.66), one can read off the magnetic permeability of the QCD vacuum,

$$\mu_{\text{vac}}^{\text{QCD}}(B) = 1 + \frac{11N_c - 2N_f}{48\pi^2} g^2 \ln \frac{\Lambda^2}{|gB|} > 1. \tag{1.69}$$

The first term in the denominator ($11N_c$) is the gluon contribution to the magnetic permeability. This term dominates over the quark contribution ($2N_f$) as long as the number of flavors N_f is less than 17 and is responsible for asymptotic freedom.

The dielectric constant as a function of distance r is then given by

$$\epsilon_{\text{vac}}^{\text{QCD}}(r) = \left. \frac{1}{\mu_{\text{vac}}^{\text{QCD}}(B)} \right|_{\sqrt{|gB|} \to 1/r}. \tag{1.70}$$

The replacement $\sqrt{|gB|} \to 1/r$ follows from the fact that ϵ and μ in (1.70) should be calculated from the same field modes: the dielectric constant $\epsilon(r)$ could be calculated by computing the vacuum energy in the presence of two static colored test particles located at a distance r from each other. In this case, the maximum wavelength of field modes that can contribute is of order r, so that

$$r \gtrsim \lambda \gtrsim \frac{1}{\Lambda}. \tag{1.71}$$

Combining eqs. (1.67) and (1.71), we identify $r = 1/\sqrt{|gB|}$ and find

$$\epsilon_{\text{vac}}^{\text{QCD}}(r) = \frac{1}{1 + \frac{11N_c - 2N_f}{24\pi^2} g^2 \ln(r\Lambda)} < 1. \tag{1.72}$$

With $\alpha_s(r_1)/\alpha_s(r_2) = \epsilon_{\text{vac}}^{\text{QCD}}(r_2)/\epsilon_{\text{vac}}^{\text{QCD}}(r_1)$ one finds to lowest order in α_s (the strong interaction coupling constant)

$$\alpha_s(r_1) = \frac{\alpha_s(r_2)}{1 + \frac{11N_c - 2N_f}{6\pi} \alpha_s(r_2) \ln\left(\frac{r_2}{r_1}\right)}. \tag{1.73}$$

Apparently, if $r_1 < r_2$ then $\alpha_s(r_1) < \alpha_s(r_2)$. The running of the coupling constant is shown in figure 1.6, where $Q \sim 1/r$ is the *momentum transfer*. The intuitive derivation given above illustrates the original field-theoretical result of [GW73].

At high momentum transfer, corresponding to short distances, the coupling constant thus becomes small and one can apply perturbation theory, see figure 1.6. There are a

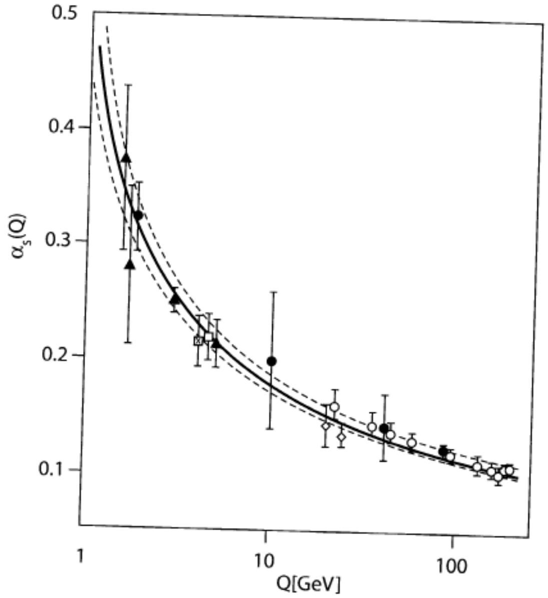

Figure 1.6 The running coupling constant $\alpha_s(Q^2)$ as a function of momentum transfer Q^2 determined from a variety of processes. The figure is from [Bet00].

variety of processes that involve high momentum scales, for example, deep inelastic scattering, Drell-Yan dilepton production, e^+e^--annihilation into hadrons, production of heavy quarks/quarkonia, high p_T hadron production, ... QCD correctly predicts the Q^2-dependence of these so-called "hard" processes, which is a great success of the theory [HM84].

While asymptotic freedom implies that the theory becomes simple and treatable at short distances, it also tells us that at large distances the coupling becomes very strong. In this regime we have no reason to believe in perturbation theory. In QED, as we have discussed above, the strong coupling regime starts at extremely short distances beyond the reach of current experiments—and this makes the "zero-charge" problem somewhat academic. In QCD, the entire physical world around us is defined by the properties of the theory in the strong coupling regime—and we have to construct accelerators to study it in the much simpler "QED–like," weak coupling limit.

We do not have to look far to find striking differences between the properties of QCD at short and large distances: the elementary building blocks of QCD—the "fundamental" fields appearing in the Lagrangian (1.59), quarks and gluons—do not exist in the physical spectrum as asymptotic states. For some reason still unknown to us, all physical states with finite energy appear to be color-singlet combinations of quarks and gluons, which are thus always "confined" at rather short distances on the order of 1 fm. This prevents us, at least in principle, from using well-developed formal S-matrix approaches based on analyticity and unitarity to describe quark and gluon interactions.

1.11 Exercises

1. a) Using the relativistic expression for the momentum-energy relation, find the de Broglie wavelength $\lambda = h/p$, for protons with kinetic energy 500 keV and 900 MeV. b) Repeat the calculation using a nonrelativistic expression for the momentum. c) Repeat (a) and (b) for electrons with the same energies.

2. For which kinetic energy does the proton have velocity equal to half that of light? Compare with the result for the electron.

3. From the uncertainty principle $\Delta p \Delta x \sim \hbar$, and the fact that a nucleon is confined within the nucleus, what can be concluded about the energies of nucleons within the nucleus?

4. What is the minimum photon energy required to dissociate the deuteron? Take the binding energy to be 2.224589 MeV.

5. Because pions had not been discovered in 1936 when Yukawa proposed the meson theory of the nuclear force, it was suggested that the muon was Yukawa's particle. What would the range of the nuclear force be if this were true?

6. Using relativistic expressions for momentum and energy conservation, show that a proton must have energy greater than 5.6 GeV to produce a proton-antiproton pair in a collision with another proton at rest.

7. Using the mass-energy relation, find the kinetic energy released in the decays in (1.8).

8. Find the threshold energies for the following reactions in the laboratory system, assuming that the initial proton is at rest:

(a) $\mathrm{p} + \mathrm{p} \;\rightarrow\; \mathrm{p} + \mathrm{p} + \pi^0$
(b) $\mathrm{p} + \mathrm{p} \;\rightarrow\; \mathrm{p} + \mathrm{n} + \pi^+$
(c) $\mathrm{p} + \mathrm{p} \;\rightarrow\; \mathrm{p} + \mathrm{p} + \pi^+ + \pi^-$
(d) $\pi^- + \mathrm{p} \;\rightarrow\; \mathrm{p} + \bar{\mathrm{p}} + \mathrm{n}$

9. Which of the following processes are absolutely forbidden?

(a) $\pi^0 + \mathrm{n} \;\rightarrow\; \pi^- + \mathrm{p}$
(b) $\mathrm{p} + \mathrm{e}^- \;\rightarrow\; \gamma + \gamma$
(c) $\mathrm{n} \;\rightarrow\; \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e$
(d) $\mathrm{n} \;\rightarrow\; \mathrm{p} + \mathrm{e}^+ + \nu_e$
(e) $\gamma + \mathrm{p} \;\rightarrow\; \bar{\mathrm{n}} + \pi^+$

10. Verify expression (1.39) for p, n, $\bar{p}$, $\bar{n}$, π^+, π^-, and π^0.

11. Assuming that the virtual pions of figure 1.1 (c and d) describe a semicircle with diameter 1 fm, find, using (1.45), the extra contribution for the magnetic dipole moment of the proton and the neutron caused by the emission of a virtual pion (note that this is a very crude model to explain the anomalous magnetic moment of protons and neutrons).

12. Which of the following processes cannot occur through the strong interaction?

(a) $K^- \rightarrow \pi^- + \pi^0$
(b) $K^- + p \rightarrow \bar{K}^0 + n$
(c) $\Xi^0 + n \rightarrow \Sigma^- + p$
(d) $\Lambda^0 + n \rightarrow \Sigma^- + p$
(e) $K^- + p \rightarrow \Lambda^0 + n$
(f) $\pi^+ + n \rightarrow K^+ + \Sigma^0$

13. Write a reaction involving the proton and the kaons (K^+, K^-, and K^0) that obeys the conservation laws and that leads to the creation of the hyperon Ω^-.

14. Use table 1.1 and show that the hyperon Ω^- decay could not have any mode governed by the strong interaction (that conserves S) that does not violate some conservation law. For example, the decay $\Omega^- \rightarrow p + 2K^- + \bar{K}^0$ conserves S but is energetically forbidden. The particle Ω^- decays, in fact, only by means of the weak force (which does not conserve S), through the branching $\Omega^- \rightarrow \Lambda + K^-$ (69%), $\Omega^- \rightarrow \Xi^0 + \pi^-$ (23%), and $\Omega^- \rightarrow \Xi^- + \pi^0$ (8%).

15. Using relation (1.39), show that the quarks up and down are members of an isospin doublet $t_z = \pm 1/2$.

16. Find the quark composition of particles in table 1.1.