3 The Nucleon-Nucleon Interaction

3.1 Introduction

The starting point for any dynamical description of a physical system is knowledge of the relevant degrees of freedom and of the interaction. In the previous chapters we have seen that nucleons are the basic components of nuclei. Their degrees of freedom are determined by the position $\mathbf{r}_i$, momentum $\mathbf{p}_i$, spin $\mathbf{s}_i$ and isospin $\boldsymbol{\tau}_i$ of the ith nucleon. For the interaction one first takes the simplest assumption that it is a two-body interaction that can be described by a potential. A further extension of the model introduces three- and many-body interactions for a deeper understanding of the many-body system. For historical reasons we will first give with a phenomenological description and later the more fundamental meson-exchange theory.

In the following section we shall examine in what way the knowledge we already have about the deuteron can help us find a reasonable description of the nucleon-nucleon interaction. We should, at the beginning, consider a group of experimental facts that indicate that the nuclear force is independent of the charge of the nucleons. This means that the force between a neutron and a proton has the same form as the force between two neutrons and also between two protons, if we subtract the Coulomb part. It also means that there is a physical quantity involved for which there is a conservation law. Such a quantity is the isospin T, defined in chapter 1. In terms of the component T_z of that quantity we can express three possibilities to build a system of two nucleons: the *di-neutron* with $T_z = -1$, the *di-proton* with $T_z = +1$, and the deuteron with $T_z = 0$. T_z denotes the sum of the Z-component of the isospin of each nucleon. Since we only have two nucleons, T cannot be larger than 1. Therefore, for both systems, the di-neutron and di-proton, T has to be equal to 1. For the deuteron with $T_z = 0$, T can be 0 or 1.

The wavefunction of a system of two nucleons can be written as the product of a space function, a spin function, and one of isospin:

$$\Psi = \psi_{\text{spa}} \chi_S^{m_s} \phi_T^{T_z}. \tag{3.1}$$

Table 3.1 Isospin wavefunction for the two-nucleon system.

Isospin wavefunction	T	T_z	Symmetry by isospin exchange
$\phi_1^1 = \pi(1)\pi(2)$	1	1	Triplet (symmetric)
$\phi_1^0 = \frac{1}{\sqrt{2}}\lvert\pi(1)\nu(2) + \pi(2)\nu(1)\rvert$	1	0	
$\phi_1^{-1} = \nu(1)\nu(2)$	1	−1	
$\phi_0^0 = \frac{1}{\sqrt{2}}\lvert\pi(1)\nu(2) - \pi(2)\nu(1)\rvert$	0	0	Singlet (antisymmetric)

We denote π as a state of the proton and ν as a state of the neutron, so that $\pi(1)\,\nu(2)$ means that the first nucleon is a proton and the second is a neutron. We can build the isospin part $\phi_{T_z}^T$ of the wavefunction of the two-nucleon system in a similar way to the case of spin, as indicated in table 3.1.

3.2 Phenomenological Potentials

In the phenomenological method one uses the appropriate functional form for the potential with a sufficient amount of parameters. The parameters are chosen so that the potential describes as closely as possible the experimental data of the NN system. There are two classes of such potentials: *local* and *nonlocal* potentials.

3.3 Local Potentials

The following general ansatz is made for the potential as a function of the relevant degrees of freedom of both nucleons:

$$V(1,2) = V(\mathbf{r}_j, \mathbf{p}_j, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_j;\ j = 1,2). \tag{3.2}$$

Symmetry and invariance properties of the Hamiltonian operator limit the general form of the interaction (see Appendix C). These properties are the requirement of invariance through translation, rotation, Galilean transformations, and particle exchange in connection with the Pauli principle, that is,

$$V(1,2) = V(2,1). \tag{3.3}$$

To account for these invariance properties one introduces the relative and center-of-mass coordinates and momenta, where the small mass difference between the neutron

Table 3.2 Tensors in two-nucleon space.

Type	Operator	Parity	Time reversal	Number
scalar	1	+	+	1
scalar	$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$	+	+	1
vector	$\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2$	+	+	3
vector	$\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2$	+	−	3
vector	$\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2$	+	−	3
tensor	$\left[\boldsymbol{\sigma}_1^{[1]} \times \boldsymbol{\sigma}_2^{[1]}\right]^{[2]}$	+	+	5
	Total number			16

and the proton is neglected:

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \qquad \mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2),$$
$$\mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2), \qquad \mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2. \tag{3.4}$$

The requirement of invariance under *translations* $\mathbf{r} \to \mathbf{r}_1 + \mathbf{a}$ leads to the condition

$$V(\mathbf{r}, \mathbf{p}, \mathbf{R}, \mathbf{P}) = V(\mathbf{r}, \mathbf{p}, \mathbf{R} + \mathbf{a}, \mathbf{P}),$$

and invariance under *Galilean transformation* $\mathbf{p}_j \to \mathbf{p}_j + \mathbf{p}_0$ implies

$$V(\mathbf{r}, \mathbf{p}, \mathbf{R}, \mathbf{P}) = V(\mathbf{r}, \mathbf{p}, \mathbf{R}, \mathbf{P} + 2\mathbf{p}_0).$$

Since $\mathbf{a}$ and $\mathbf{p}_0$ can take any values, these relations mean that V cannot depend on $\mathbf{R}$ and $\mathbf{P}$, so it can only possess the form

$$V(1,2) = V(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}_j, \boldsymbol{\tau}_j;\ j = 1,2). \tag{3.5}$$

Next we study the *rotational invariance* property. This determines the structure of the spin degrees of freedom. Any function $f(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$ represents a 4×4 matrix in the space of two-nucleon spin that can be spanned by a linear combination of 16 matrices. These can be classified by their tensor properties, as shown in table 3.2. The indices [1] and [2] refer the coupling scheme of two tensor operators $T_1^{[L_1]}$ and $T_2^{[L_2]}$ into a new operator

$$T_{[M]}^{[L]} = \left[T_1^{[L_1]} \times T_2^{[L_2]}\right]_{[M]}^{[L]} = \sum_{M_1 M_2} (L_1 M_1 L_2 M_2 | LM) T_{[M_1]}^{[L_1]} T_{[M_2]}^{[L_2]}.$$

The vector operator $\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2$ in the third row of table 3.2 does not carry the similar notation, $\left[\boldsymbol{\sigma}_1^{[1]} \times \boldsymbol{\sigma}_2^{[1]}\right]^{[1]}$, for the sake of simplicity. For more details on tensor operators, see Appendix B.

Table 3.3 Tensors build from r and p.

Type	Operator	Parity	Time reversal
scalar	$\mathbf{r}^2$	+	+
scalar	$\mathbf{p}^2$	+	+
scalar	$\mathbf{r}\cdot\mathbf{p}$	+	−
vector	$\mathbf{r}$	−	+
vector	$\mathbf{p}$	−	−
vector	$\mathbf{r}\times\mathbf{p}$	+	−
tensor	$[\mathbf{r}^{[1]}\times\mathbf{r}^{[1]}]^{[2]}$	+	+
tensor	$[\mathbf{p}^{[1]}\times\mathbf{p}^{[1]}]^{[2]}$	+	+
tensor	$[\mathbf{r}^{[1]}\times\mathbf{p}^{[1]}]^{[2]}$	+	−

When one constructs the potentials in terms of these linear combinations, one must be sure that the result is a scalar and that the symmetries under particle exchange, parity, and time reversal are observed. This means that one has to combine the vector and tensor symmetry operators with the corresponding vector and tensor operators obtained from **r** and **p**. The possible operators obtained in this way are shown in table 3.3.

Due to consideration of symmetry and invariance properties, only the following vector-vector and tensor-tensor combinations are possible:

(a) Vector-vector: spin-orbit operator

$$\mathbf{L}\cdot\mathbf{S}=\frac{1}{2}(\mathbf{r}\times\mathbf{p})\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2). \tag{3.6}$$

(b) Tensor-tensor:

$$[\mathbf{r}^{[1]}\times\mathbf{r}^{[1]}]^{[2]}\cdot\left[\boldsymbol{\sigma}_1^{[1]}\times\boldsymbol{\sigma}_2^{[1]}\right]^{[2]}=(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})-\frac{1}{3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,r^2,$$

$$[\mathbf{p}^{[1]}\times\mathbf{p}^{[1]}]^{[2]}\cdot\left[\boldsymbol{\sigma}_1^{[1]}\times\boldsymbol{\sigma}_2^{[1]}\right]^{[2]}=(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{p})-\frac{1}{3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,p^2,$$

$$\left[\boldsymbol{\sigma}_1^{[1]}\times\boldsymbol{\sigma}_2^{[1]}\right]^{[2]}\cdot[\mathbf{r}^{[1]}\times\mathbf{p}^{[1]}]^{[2]}\,\mathbf{r}\cdot\mathbf{p}. \tag{3.7}$$

Instead of the last tensor operator, one uses the equivalent square of the spin-orbit operator $(\mathbf{L}\cdot\mathbf{S})^2$.

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invaria

and time reversal, is given by

$$V(1,2) = V_C + V_S(\sigma_1 \cdot \sigma_2) + V_T S_{12}(\mathbf{r}) + V'_T S_{12}(\mathbf{p}) + V_{LS}\mathbf{L}\cdot\mathbf{S} + V_Q(\mathbf{L}\cdot\mathbf{S})^2. \tag{3.8}$$

where the operator S_{12} is given by

$$S_{12} = 3\left(\sigma_1 \cdot \frac{\mathbf{r}}{r}\right)\left(\sigma_2 \cdot \frac{\mathbf{r}}{r}\right) - (\sigma_1 \cdot \sigma_2). \tag{3.9}$$

In (3.8) the quantities V_α with $\alpha \in \{C, S, T, T', LS; Q\}$ are scalar functions of the remaining scalars r^2, p^2, and $(\mathbf{r}\cdot\mathbf{p})^2$. Due to the relation

$$(\mathbf{r}\cdot\mathbf{p})^2 = r^2p^2 - L^2, \tag{3.10}$$

one chooses instead the variables r^2, p^2, and L^2 as the independent ones. One must also be sure that the total V is a Hermitian operator.

As a last point, we have to consider the isospin dependence of the interaction. The experimental data indicate that the NN interaction is approximately independent of the charge state of the nucleons, that is, of nn, pp, or np. In fact, the states ϕ_1^1 (di-proton) and ϕ_1^{-1} (di-neutron) discussed in the last chapter constitute, together with ϕ_1^0, a triplet in the isospin space. Now we want to know if some member of that triplet can be part of a bound state of the two particles. To show that this is not possible, let us examine the ground state of the deuteron. We saw that this state has $J = 1$, $S = 1$, and $l = 0$. The last value indicates that the space part is symmetrical and that $S = 1$ also corresponds to a symmetrical spin part. As Ψ in (3.1) should be antisymmetric, $\phi_T^{T_z}$ should also be antisymmetric for the ground state of the deuteron, and the isospin wavefunction of that state can only be ϕ_0^0. The function ϕ_1^0 is, therefore, the isospin wavefunction of an excited state of the deuteron. But we know experimentally that this state is not bound. As the nuclear force does not depend on the charge, the absence of a bound state for ϕ_1^0 should be extended to ϕ_1^1 and ϕ_1^{-1}. This last result exhibits that *the bound proton-proton or neutron-neutron system does not exist.*

But, how can states with $T = 1$ and $T = 0$ correspond to different energies if the nuclear forces are independent of the charge (isospin)? This is due to the dependence of the nuclear force on the spin. To each group of isospin states is associated a different orientation for the spins, so that to each group correspond different energies. The dependence of the nuclear force on the spin has a connection with the fact that there is no state for the deuteron other than the ground state (triplet spin). The force between the proton and the neutron when they have antiparallel spins (singlet) is smaller than when they have parallel spins (triplet), not strong enough to form a bound state. This force has a value just a little below that necessary to produce a bound state.

One can formally account for isospin independence by using the commutator property $[H, T_\pm] = 0$, where T is the total isospin operator $\mathbf{T} = \mathbf{t}_1 + \mathbf{t}_2$. Together with the charge conservation property $[H, T_z] = 0$, it follows that $[H, \mathbf{T}^2] = 0$, that is, an invariance under complete rotations in the isospin space. In other words, the interaction between a neutron and a proton cannot be different from that in any coherent superposition of both. Under these assumptions for isospin invariance, the functions V_α in (3.8) must be scalars in the

isospin space in the form

$$V_a = V_{a0} + V_{a1}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2. \tag{3.11}$$

Sometimes it is convenient to describe the spin and isospin dependence of the NN interaction in terms of projection operators. We will show in the following that the terms of (3.8) can be derived in a more physically transparent way. For example, the spin part of the interaction can be written as

$$V_\sigma(r) \cdot \frac{1}{2}(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \equiv V_\sigma(r) P_\sigma, \tag{3.12}$$

where $V_\sigma(r)$ describes the radial dependence and the operator $P_\sigma = \frac{1}{2}(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$ has the expected values $+1$ for the triplet state and -1 for the singlet state. This can be shown starting from the vector $\mathbf{S} = \frac{\hbar}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$. Since $S^2 = \frac{\hbar^2}{4}(\sigma_1^2 + \sigma_2^2 + 2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$, then

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = \frac{1}{2}\left(-\sigma_1^2 - \sigma_2^2 + \frac{4S^2}{\hbar^2}\right); \tag{3.13}$$

the eigenvalues $\hbar^2 S(S+1)$ of S^2 are $+2\hbar^2$ for the triplet state ($S = 1$) and 0 for the singlet state. The eigenvalues of $\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$ are equal to 3, so that the eigenvalues of $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$ are equal to $+1$ for the triplet state and -3 for the singlet state, resulting in the expected values of P_σ predicted above.

P_σ is known as the *Bartlett potential* or *spin exchange potential* since, if we use for the spin functions similar to those given in table 3.1, we shall obtain that the operation of spin exchange is equivalent to multiplication by a factor $+1$ for the triplet state and a factor -1 for the singlet state.

If one assumes that the nuclear force depends on the parity of the wavefunction that describes the two particles, then a way of expressing that dependence is by means of a term of the form

$$V_r(r) P_r, \tag{3.14}$$

referred to as the *Majorana potential*, which contains the operator P_r that exchanges the space coordinates of the two particles. The eigenvalues of P_r are $+1$ and -1, if the wavefunction is even or odd, respectively.

The isospin dependence of the interaction can also be defined by the quantity

$$V_t(r) \cdot \frac{1}{2}(1 + \mathbf{t}_1 \cdot \mathbf{t}_2) \equiv V_t(r) P_t, \tag{3.15}$$

where the operator P_t changes the isospin of the two particles. The antisymmetry of the total wavefunction implies that P_t is not independent of P_σ and P_r, embodied in the relation

$$P_t = -P_\sigma P_r, \tag{3.16}$$

which can be verified easily by the application of both sides to (3.1). The operator $P_\sigma P_r$ is known as the *Heisenberg operator*.

Gathering the terms presented up to now, we can write the expression that represents the central part of the nucleon-nucleon potential:

$$V_C(r) = V_W(r) + V_r(r) P_r + V_\sigma(r) P_\sigma + V_t(r) P_t, \tag{3.17}$$

The existence of tensor interactions becomes necessary to explain certain experimental results. One of them is the absence of a well-defined value for l, represented by the disturbance of a state $l = 2$ caused in the ground state $l = 0$ of the deuteron, (2.80). This forces us to think in terms of a noncentral nucleon-nucleon interaction potential, $V(\mathbf{r})$, since a central potential $V(r)$ conserves angular momentum and has l as a good quantum number. This explains the existence of the S_{12} term in (3.8).

It is common to describe the action of those noncentral forces by a function of the angles between the spin vectors of the neutron and of the proton and of the radial vector r that separates them. Such a potential is known as the *tensor potential*. The candidate functions to represent the tensor potential should, as a first requirement, be a scalar. Thus, with $\mathbf{u}_r$ the unitary vector in the direction $\mathbf{r}$, products of the type $\boldsymbol{\sigma}_1 \cdot \mathbf{u}_r$, $\boldsymbol{\sigma}_2 \cdot \mathbf{u}_r$, and $(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \cdot \mathbf{u}_r$ must be rejected as pseudoscalars, that is, they change sign under reflection of the system of coordinates. Powers of those expressions are useless since we have, for example, $(\boldsymbol{\sigma} \cdot \mathbf{u}_r)^2 = 1$ and $(\boldsymbol{\sigma} \cdot \mathbf{u}_r)^3 = \boldsymbol{\sigma} \cdot \mathbf{u}_r$. In this situation, the simplest form of scalars we are looking for is $(\boldsymbol{\sigma}_1 \cdot \mathbf{u}_r)(\boldsymbol{\sigma}_2 \cdot \mathbf{u}_r)$[1]. This expression is usually modified to satisfy the condition that the average value about all directions is zero. Since we know that the average of $(\mathbf{A} \cdot \mathbf{u}_r)(\mathbf{B} \cdot \mathbf{u}_r)$ is $\frac{1}{3}\mathbf{A} \cdot \mathbf{B}$, we define the potential tensor as in (3.9).

For the singlet state, $\boldsymbol{\sigma}_1 = -\boldsymbol{\sigma}_2$, where it follows that $(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) = -\sigma_1^2 = -3$, and $(\boldsymbol{\sigma}_1 \cdot \mathbf{u}_r)(\boldsymbol{\sigma}_2 \cdot \mathbf{u}_r) = -(\boldsymbol{\sigma}_1 \cdot \mathbf{u}_r)^2 = -1$. Thus, for the singlet state, $S_{12} = 0$, the tensor force is zero. This is an expected result since there is no preferential direction for the singlet state.

The terms described above are all characteristic of a *local potential*, an expression that denotes a potential that is perfectly defined at each point r of the space. Potentials dependent on momentum are, on the other hand, examples of potentials that do not only depend on one point and are called *nonlocal*. Among these, it is common to include in the nuclear potential a term of the form $V_{LS}(r)\mathbf{L} \cdot \mathbf{S}$. Thus, (3.8) is not completely local, as it is linear in p, and is known as the *spin-orbit interaction*. This interaction can be observed, for example, in the scattering of polarized protons by a spinless target nucleus (figure 3.1). Depending on which direction the proton travels, the spin $\mathbf{S}$ and the angular momentum $\mathbf{L}$ can be parallel or antiparallel. Therefore, the term $V_{LS}(r)\mathbf{L} \cdot \mathbf{S}$ in the potential has a scalar product which is some times positive, other times negative. This leads to an asymmetry in the scattering cross section.

To establish the form of the unknown functions contained in (3.8), one adopts the approach that this potential describes correctly the experimental observations on nucleon-nucleon scattering, or the properties of certain nuclei, as for instance the deuteron. The values of these functions should be adjusted in such a way that they satisfy the approach above; we shall get a *phenomenological potential*. Phenomenological potentials are broadly employed, not only in the construction of nucleon-nucleon forces, but also in the interaction of complex nuclei, where the participation of the individual nucleons becomes extremely difficult to describe.

Phenomenological parameterizations for the nuclear potential possess attractive and repulsive components. At great distances they are reduced to the *one-pion exchange potential*

[1] The alternative $(\boldsymbol{\sigma}_1 \times \mathbf{u}_r)(\boldsymbol{\sigma}_2 \times \mathbf{u}_r)$ is not relevant, being a linear combination of $(\boldsymbol{\sigma}_1 \cdot \mathbf{u}_r)(\boldsymbol{\sigma}_2 \cdot \mathbf{u}_r)$ and $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$.

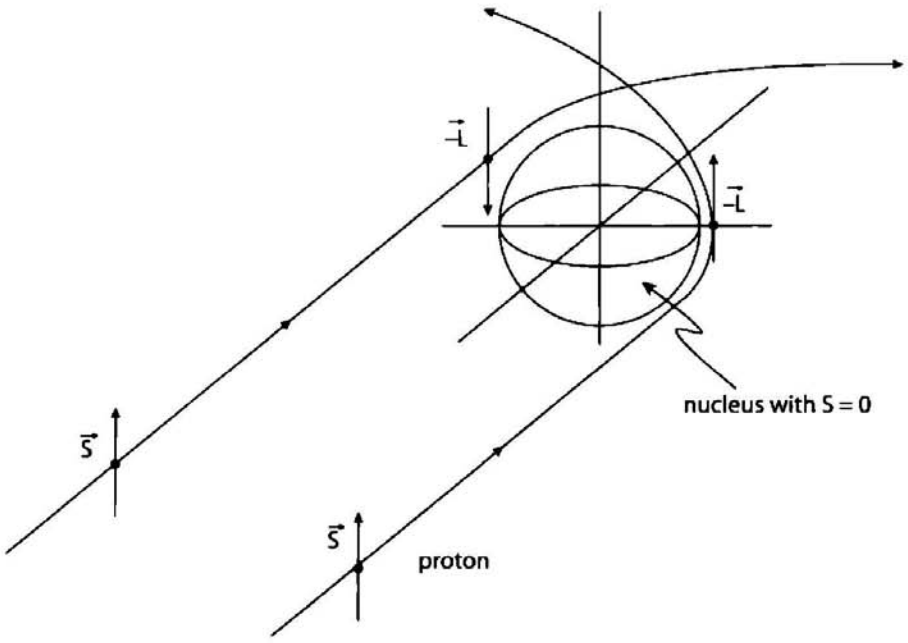

Figure 3.1 Scattering of polarized protons.

(OPEP), while at small distances they possess an extremely repulsive part. That repulsive part is usually referred to as the "hard core," with $V(r) \to \infty$ for $r < r_c \cong 0.4$ fm. Some authors use a repulsive potential that goes to infinity only for $r \to 0$. These potentials are known as "soft core" potentials. The most popular of these potentials is the *Reid soft-core* potential [Re68]. It has the form

$$V = V_c(\mu r) + V_{12}(\mu r)S_{12} + V_{LS}(\mu r)\mathbf{L}\cdot\mathbf{S}, \tag{3.18}$$

where

$$V_c(x) = \sum_{n=1}^{\infty} a_n \frac{e^{-nx}}{x}, \quad V_{LS}(x) = \sum_{n=1}^{\infty} c_n \frac{e^{-nx}}{x}, \tag{3.19}$$

and

$$V_{12}(x) = \frac{b_1}{x}\left[\left(\frac{1}{3}+\frac{1}{x}+\frac{1}{x^2}\right)e^{-x} - \left(\frac{b_0}{x}+\frac{1}{x^2}\right)e^{-b_0 x}\right] + \sum_{n=2}^{\infty} b_n \frac{e^{-nx}}{x}. \tag{3.20}$$

The constants are different for all values of T, S, and L. Only a_1, b_1, and c_1 are given in order to reproduce the OPEP potential at great distances. For $l > 2$, the Reid potential is replaced by the OPEP potential. The Reid potential is quite realistic and describes well, within its range of validity, the properties of a system of two nucleons.

3.3.1 Nonlocal potential

The most general form of a potential, including local and nonlocal characteristics, can be represented by an integral operation of the form

$$\langle \mathbf{r}|\widehat{V}|\psi\rangle = \int d^3r' \langle \mathbf{r}|\widehat{V}|\mathbf{r}'\rangle\langle \mathbf{r}'|\psi\rangle = \int d^3r' V(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}'). \tag{3.21}$$

This potential leads to an integro-differential Schrödinger equation. The special case of a local potential is represented by the diagonal form

$$V(\mathbf{r},\mathbf{r}') = V(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}'). \tag{3.22}$$

In other words, a local potential is such that the relation

$$\langle \mathbf{r}|\widehat{V}|\psi\rangle = V(\mathbf{r})\psi(\mathbf{r}) \tag{3.23}$$

is valid. It means that the action of the interaction at the point $\mathbf{r}$ only depends on the value of $\psi(\mathbf{r})$ at that point. Thus, a momentum-dependent potential $V(\mathbf{r},\mathbf{p})$ does not belong to the family of local potentials because the dependence on $\mathbf{p}$ implies a dependence of the potential on the neighborhood of $\mathbf{r}$. In fact, at the end of this section we will see that there is an equivalence between the dependence on momentum (or velocity) and nonlocal potentials.

For $V(\mathbf{r},\mathbf{r}')$ one can again use a few symmetry invariance conditions to fix the form of the potential. We will not take this route here. Instead, we will describe a simpler class of potentials called *separable potentials*, which can be represented by

$$V(\mathbf{r},\mathbf{r}') = f^*(\mathbf{r})f(\mathbf{r}'). \tag{3.24}$$

One obtains for this class of potentials

$$\langle \mathbf{r}|\widehat{V}|\psi\rangle = f^*(\mathbf{r})\int d^3r' f(\mathbf{r}')\psi(\mathbf{r}'), \tag{3.25}$$

which leads to a great simplification of the Schrödinger equation.

The main source of nonlocality of the NN interaction is the size of the nucleon: the fact that it has an internal structure and dynamics. The relativistic effects, e.g., retardation effects, also lead to nonlocality.

Let us calculate the matrix element of V between any states in the coordinate representation

$$\langle \phi|\widehat{V}|\psi\rangle = \int d^3r' d^3r'' \langle \phi|\mathbf{r}''\rangle V(\mathbf{r},\mathbf{r}')\langle \mathbf{r}'|\psi\rangle. \tag{3.26}$$

With the help of the transformation of the integration variables

$$\mathbf{r} = \frac{1}{2}(\mathbf{r}''+\mathbf{r}'), \quad \boldsymbol{\rho} = \mathbf{r}''-\mathbf{r}',$$

and the Taylor expansion, represented by the translation operator, $D(\mathbf{r})$, described by (A.23) of Appendix A,

$$\langle \mathbf{r}'|\psi\rangle = \left\langle \mathbf{r}-\frac{1}{2}\boldsymbol{\rho}\middle|\psi\right\rangle = \left\langle \mathbf{r}\middle|D\left(\frac{1}{2}\boldsymbol{\rho}\right)\middle|\psi\right\rangle. \tag{3.27}$$

one can reduce the matrix element to the form

$$\langle \phi|\widehat{V}|\psi\rangle = \int d^3r \langle \phi|\mathbf{r}\rangle \widetilde{V}(\mathbf{r},\mathbf{p})\langle \mathbf{r}|\psi\rangle. \tag{3.28}$$

where

$$\widetilde{V}(\mathbf{r},\mathbf{p}) = \int d^3\rho\, e^{-\frac{i}{2}\boldsymbol{\rho}\cdot\mathbf{p}} V\left(\mathbf{r}+\frac{1}{2}\boldsymbol{\rho},\mathbf{r}-\frac{1}{2}\boldsymbol{\rho}\right) e^{-\frac{i}{2}\boldsymbol{\rho}\cdot\mathbf{p}}$$
$$= \int d^3\rho\, V\left(\mathbf{r}+\frac{1}{2}\boldsymbol{\rho},\mathbf{r}-\frac{1}{2}\boldsymbol{\rho}\right) + \frac{i}{2}\int d^3\rho \left\{\boldsymbol{\rho}\cdot\mathbf{p}, V\left(\mathbf{r}+\frac{1}{2}\boldsymbol{\rho},\mathbf{r}-\frac{1}{2}\boldsymbol{\rho}\right)\right\} + \cdots \tag{3.29}$$

with $\mathbf{p} = -i\nabla$ and $\{\ \}$ meaning the anticommutator. One alternative form is

$$\widetilde{V}(\mathbf{r},\mathbf{p}) = \frac{1}{2}\int d^3\rho \left[V(\mathbf{r},\mathbf{r}-\boldsymbol{\rho})\, e^{-i\boldsymbol{\rho}\cdot\mathbf{p}} + \text{h.c.}\right]. \tag{3.30}$$

Now let us assume the opposite situation, in which a momentum-dependent potential $V(\mathbf{r},\mathbf{p})$ is given. One can always bring it to the form

$$V(\mathbf{r},\mathbf{p}) = v(\mathbf{r},\mathbf{p}) + v^{\dagger}(\mathbf{r},\mathbf{p}), \tag{3.31}$$

where in $v(\mathbf{r},\mathbf{p})$ all $\mathbf{p}$-terms are on the right of the $\mathbf{r}$ operators. For such matrix elements one gets

$$\langle \mathbf{r}'|v(\mathbf{r},\mathbf{p})|\mathbf{r}''\rangle = \int d^3r''\delta(\mathbf{r}-\mathbf{r}'')\, v(\mathbf{r}'',\mathbf{p}'')\,\delta(\mathbf{r}'-\mathbf{r}'')$$
$$= \frac{1}{(2\pi)^3}\int d^3r''\delta(\mathbf{r}-\mathbf{r}'')\, v(\mathbf{r}'',\mathbf{p}'')\int d^3k\, e^{i\mathbf{k}\cdot(\mathbf{r}''-\mathbf{r}')}$$
$$= \frac{1}{(2\pi)^3}\int d^3r''\delta(\mathbf{r}-\mathbf{r}'')\int d^3k\, v(\mathbf{r}'',\mathbf{k})\, e^{i\mathbf{k}\cdot(\mathbf{r}''-\mathbf{r}')}$$
$$= \frac{1}{(2\pi)^3}\int d^3k\, v(\mathbf{r},\mathbf{k})\, e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}, \tag{3.32}$$

and consequently nonlocal potential

$$\langle \mathbf{r}'|V|\mathbf{r}''\rangle = \frac{1}{(2\pi)^3}\int d^3k \left[v(\mathbf{r},\mathbf{k})\, e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} + \text{h.c.}\right]. \tag{3.33}$$

The construction of an equivalent $\mathbf{p}$-dependent potential V leads naturally back to the original potential, as one can easily verify by using (3.30).

3.4 Meson Exchange Potentials

3.4.1 *Yukawa and Van der Waals potentials*

Another way to attack the nuclear force problem is to be found in analyzing the meson exchange processes directly. The simplest exchange potential is due to the exchange of just one pion. But, only the long distance part of the potential can be explained in that way. Since the pion has spin zero, its wavefunction should be described by the Klein-Gordon

equation

$$\left(\nabla^2 - \frac{m_\pi^2 c^2}{\hbar^2}\right)\Phi = \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2}. \tag{3.34}$$

Performing a separation of variables, we obtain a time-independent wave equation when the total energy of the pion is equal to 0 (binding energy equal to the rest mass),

$$(\nabla^2 - \mu^2)\phi = 0. \tag{3.35}$$

where

$$\mu = \frac{m_\pi c}{\hbar}. \tag{3.36}$$

An acceptable solution for (3.35) is

$$\phi = g\frac{e^{-\mu r}}{r}, \tag{3.37}$$

in which g is a constant that has the same role as the charge in the case of electrostatics, where the potential that results from the interaction between two equal charges is $qV = q^2/r$. That interaction is due to the continuous exchange of virtual photons between the charges.

We can assume that the potential between two nucleons is proportional to the wavefunction of the pion, that is, to the probability amplitude that the emitted pion finds itself close to the other nucleon. We thus find the *Yukawa potential*

$$V = g^2\frac{e^{-\mu r}}{r}, \tag{3.38}$$

where we have used the factor g^2, in analogy with electrostatics. The potential above decays exponentially, and its range can be estimated by

$$R \cong \frac{1}{\mu} = \frac{\hbar}{m_\pi c} \cong 0.7 \text{ fm}. \tag{3.39}$$

in agreement with the result obtained in section 1.2, where the uncertainty principle was used. The force field between two protons, or two neutrons, can only be produced by the exchange of neutral pions. Between a proton and a neutron the exchange can be done by means of charged pions.

It is a well-established experimental fact that the nuclear force is strongly repulsive at very short distances and the form of the central part of the nuclear potential should be given, schematically, as in figure 3.2. The potential well, that is, its attractive part of medium range, can be described by the exchange of two pions. It is interesting to observe that this part of the potential is created similarly to the Van der Waals force between two molecules (figure 3.4).

From QCD, the fundamental theory of strong interactions, we know that nucleons are colorless objects, that is, when they are looked upon from the outside, no net color charge is visible. The same is true for neutral nonpolar molecules that contain equal positive and negative electromagnetic charges distributed with no net shift, and hence no net charge or dipole moment. However, when two molecules approach one another, the charges become polarized and each molecule acquires a nonzero dipole moment. Then

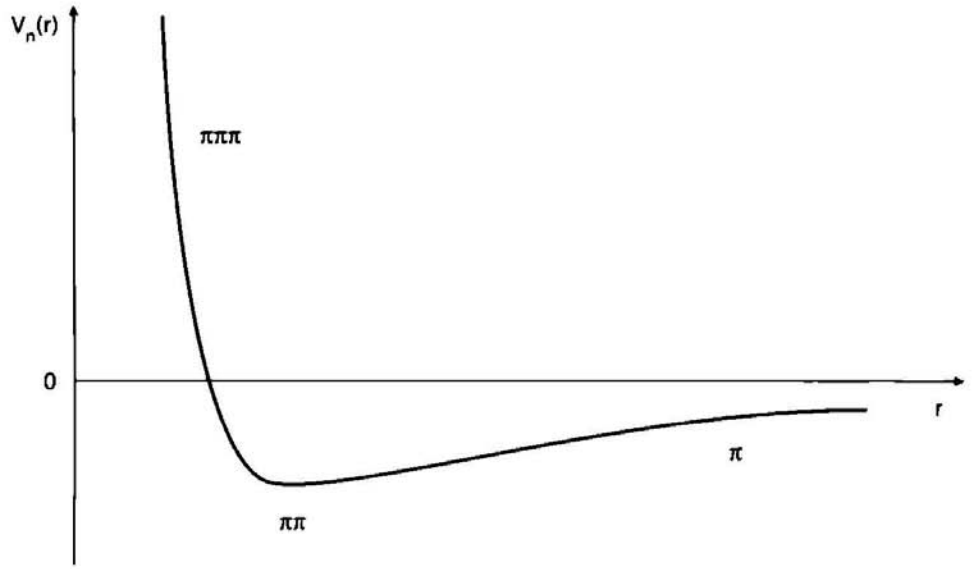

Figure 3.2 Sketch of the nucleon-nucleon potential. See section 3.4.3 for a fuller interpretation of the form of the potential.

the leading-order interaction energy between molecules equals $V(\mathbf{r}) = -2\mathbf{E}(\mathbf{r}) \cdot \mathbf{d}(\mathbf{r})$, where $\mathbf{E}(\mathbf{r})$ is the average electric field felt by one of the molecules when the second one is located at r, and $\mathbf{d}(\mathbf{r})$ is its induced dipole moment. Assuming that the induced dipole moment $\mathbf{d}(\mathbf{r})$ depends linearly on the electric field, and knowing that the electric field created by a dipole decreases as $1/r^3$, we obtain immediately that $V(\mathbf{r}) \sim -1/r^6$, which gives the well-known *Van der Waals potential.* At intermediate and small distances, polarization effects become stronger, and higher induced multipole moments begin to be active; however, we can model these effects by a phenomenological term that is equal to the square of the Van der Waals term. One thus obtains the Lennard-Jones potential,

$$V_{LJ}(r) = 4E_{p,0}\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right], \tag{3.40}$$

where $E_{p,0}$ and σ are parameters fitted to data.

Figure 3.3 shows a comparison of the NN Argonne v18 potential in the $^1S_0$ channel, with the Lennard-Jones potential between two O_2 molecules ($E_{p,O} = 10$ meV and $\sigma = 0.358$ nm). The *Argonne v18* [WSS95] is a phenomenological potential, very successful in describing NN scattering properties. In figure 3.3 both the v18 and the molecular potential are drawn in the same figure with two abscissas (the lower one for O_2-O_2, the upper one for the NN potential) and two ordinates (the left one for O_2-O_2, the right one for NN). Scales on the abscissas were fixed so as to put the minima of potentials at the same point, and differ by a factor of about 0.5×10^8, while scales on the ordinates differ by a factor of 10^{10}.

Despite the tremendous differences in scales, the two both potentials are qualitatively very similar. Amazingly, the electromagnetic molecule-molecule potential is stiffer at the minimum than the neutron-neutron "strong" potential. In this respect, it is fully justified to put the word "strong" in quotation marks—this potential is not strong at all! Both potentials exhibit a very strong repulsion at short distances—the so-called hard core (the

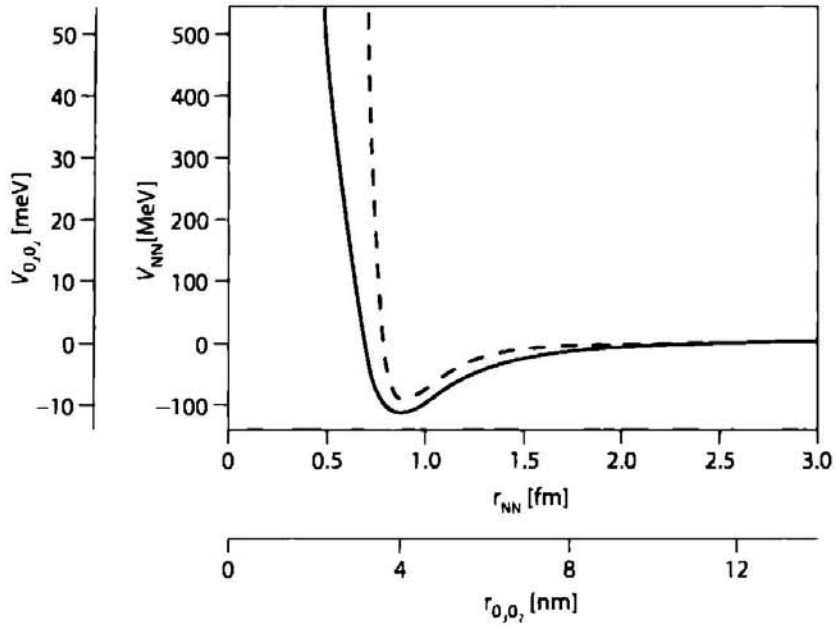

Figure 3.3 The NN potential in MeV (solid curve) in the $^1S_0$ channel as a function of the NN distance in fm compared to the O_2-O_2 molecular potential in meV (dashed curve) as a function of the distance of separation in nanometers (outer axes).

O_2-O_2 repulsion is stronger!). At large distances, there appears a weak attraction (the NN attraction vanishes more slowly despite the exponential form of the OPEP potential). Neither of the potentials is strong enough to bind the constituents into a composite object.

The analogy between the "strong" NN force and the electromagnetic molecule-molecule force is extremely instructive. First of all, we can demystify the OPEP potential in the sense that the exchange of real particles (pions) is, in fact, not its essential element. The OPEP potential is a remnant of a tool (quantum field theory) that one uses to derive it, but on a deeper level it is an effect of the color force between color-polarized composite particles. After all, nobody wants to interpret the dipole-dipole intermolecular O_2-O_2 force by an exchange of a "particle." This force can be understood in terms of a more fundamental interaction—the Coulomb force. Second, although the asymptotic large-distance leading-order behavior of both potentials can fairly easily be derived, at intermediate and small distances the interaction becomes very complicated. This is not a reflection of complications on the level of fundamental forces (color or electromagnetic), but a reflection of the complicated polarization effects that take place when composite objects are put close to one another. Moreover, these polarization effects have per se quantum character, because the fermionic constituents do not like being put close to one another—the Pauli exclusion principle creates additional polarization and repulsion effects. And third, it is obvious that at small distances effects must appear that are of a three-body character. Namely, when three O_2 molecules approach each other (e.g., in liquid oxygen), the basic assumption that they polarize one another only in pairs does not hold. There are certainly polarization effects that depend on explicit positions of all three of them. Similarly, when three nucleons

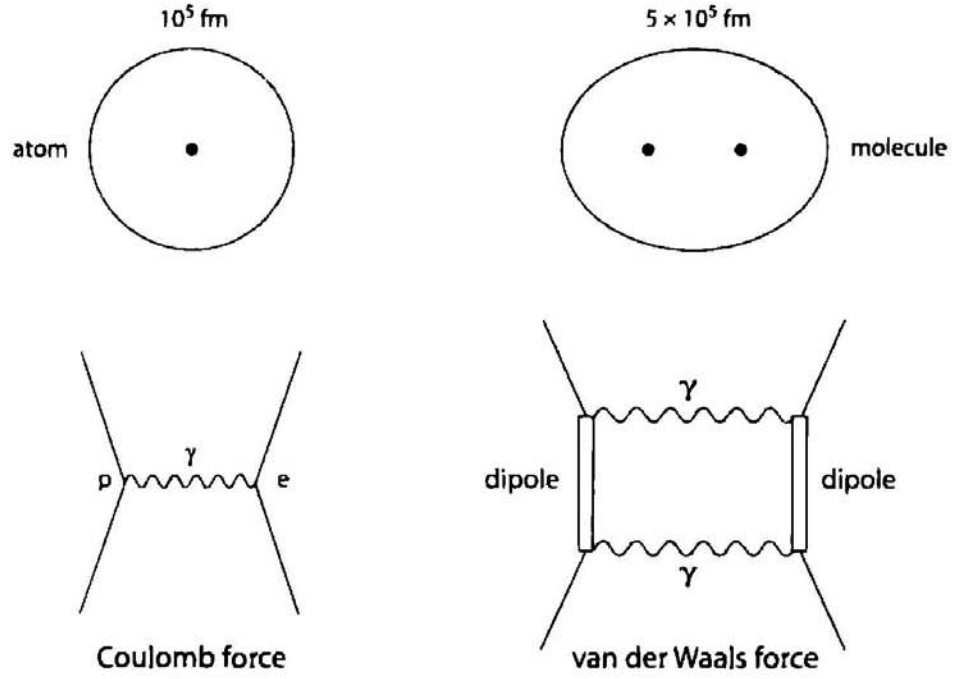

Figure 3.4 The Coulomb force between protons and electrons in an atom can be described in quantum electrodynamics by the exchange of one photon, and the Van der Waals force between atoms and molecules by the exchange of more than one photon.

approach each other within the nucleus, their quark-gluon magma becomes polarized in a fairly complicated way, which on the level of potential energy (total-interaction energy) reveals additional terms depending on the three positions simultaneously; this gives the *three-body NNN force*.

3.4.2 *Field theory picture*

If we stick to the quantum field theory picture in which forces are described by the exchange of particles, the molecular binding can be described by the exchange of two photons. The first photon, emitted by molecule 1, induces an electric dipole in molecule 2, and this dipole emits a virtual photon that induces another electric dipole in molecule 1. The interaction between the two dipoles gives rise to the Van der Waals force. The pions take the place of the photons in the case of the nuclear forces. The production of an electric dipole is similar to the excitation of a nucleon to a Δ-resonance. In this way, the nuclei are bound by a type of Van der Waals force (figure 3.4).

A more detailed comparison between the theory of pion (or other types of mesons) exchange and the electrostatic Coulomb field is necessary to account for the spin (and isospin) dependence of the interaction. Since the Coulomb field is represented by a static scalar field ϕ_γ, a massive static pion field can be represented by a static (pseudoscalar) isovector field $\boldsymbol{\phi}_\pi$. The relationships between the equations applicable to the static fields ϕ_γ and $\boldsymbol{\phi}_\pi$ are shown in table 3.4. Following this table, the static field generated by one

Table 3.4 Comparison between the equations for the static Coulomb and meson fields.

	Photon exchange	Meson exchange
Scalar field	ϕ_γ	ϕ_π
Static field eq.	$\Delta\phi_\gamma(\mathbf{x}) = -4\pi\rho_\gamma(\mathbf{x}, \mathbf{r}_1)$	$(\Delta - m_\pi^2)\,\phi_\pi(\mathbf{x}) = -\rho_\pi(\mathbf{x}, \mathbf{r}_1)$
Point charge	$\rho_\gamma(\mathbf{x}, \mathbf{r}_1) = q_1\delta(\mathbf{x} - \mathbf{r}_1)$	$\rho_\pi(\mathbf{x}, \mathbf{r}_1) = \frac{f_\pi}{m_\pi}\boldsymbol{\tau}_1\boldsymbol{\sigma}_1 \cdot \nabla_{\mathbf{r}_1}\delta(\mathbf{x} - \mathbf{r}_1)$
Solution	$\phi_\gamma(\mathbf{x}, \mathbf{r}_1) = \frac{q_1}{\lvert\mathbf{r}_1 - \mathbf{r}_2\rvert}$	$\phi_\pi(\mathbf{x}, \mathbf{r}_1) = \frac{f_\pi}{4\pi m_\pi}\boldsymbol{\tau}_1\boldsymbol{\sigma}_1 \cdot \nabla_{\mathbf{r}_1}\frac{\exp\lvert -m_\pi\lvert\mathbf{r}_1 - \mathbf{r}_2\rvert\rvert}{\lvert\mathbf{r}_1 - \mathbf{r}_2\rvert}$

particle leads to a potential energy due to the interaction with a second particle given by

$$V_\gamma(\mathbf{r}_1, \mathbf{r}_2) = \int d^3x\, \rho(\mathbf{x}, \mathbf{r}_2)\, \phi_\gamma(\mathbf{x}, \mathbf{r}_1) = \frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|},$$

$$\begin{aligned} V_\pi(\mathbf{r}_1, \mathbf{r}_2) &= \int d^3x\, \rho_\pi(\mathbf{x}, \mathbf{r}_2) \cdot \phi_\pi(\mathbf{x}, \mathbf{r}_1) \\ &= -\frac{f_\pi^2}{4\pi m_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\boldsymbol{\sigma}_1 \cdot \nabla_1\boldsymbol{\sigma}_2 \cdot \nabla_2\frac{\exp|-m_\pi|\mathbf{r}_1 - \mathbf{r}_2|}{|\mathbf{r}_1 - \mathbf{r}_2|}. \end{aligned} \tag{3.41}$$

One thus obtains in the case of the static pion field

$$V_\pi(\mathbf{r}) = \frac{f_\pi^2}{4\pi m_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\boldsymbol{\sigma}_1 \cdot \nabla_1\boldsymbol{\sigma}_2 \cdot \nabla_2\frac{\exp|-m_\pi r|}{r}, \tag{3.42}$$

which is the so-called one-pion exchange potential (OPEP) with coupling constant $f_\pi^2/4\pi$. This coupling has an empirical value given by $f_\pi^2/4\pi \simeq 0.08$. The spin operator can be written in terms of a scalar and a tensor operator by means of

$$\begin{aligned} \boldsymbol{\sigma}_1 \cdot \mathbf{a}_1\boldsymbol{\sigma}_2 \cdot \mathbf{a}_2 &= \frac{1}{3}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\,\mathbf{a}^2 + \sqrt{5}\left[\left[\boldsymbol{\sigma}_1^{[1]} \times \boldsymbol{\sigma}_2^{[1]}\right]^{[2]} \cdot \left[\mathbf{a}^{[1]} \times \mathbf{a}^{[1]}\right]^{[2]}\right]^{[0]} \\ &= \frac{1}{3}\left(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\,\mathbf{a}^2 + S_{12}(\mathbf{a})\right). \end{aligned}$$

where

$$S_{12}(\mathbf{a}) = 3\boldsymbol{\sigma}_1 \cdot \mathbf{a}\boldsymbol{\sigma}_2 \cdot \mathbf{a} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\mathbf{a}^2$$

is the tensor operator (see equation 3.9). Replacing $\mathbf{a}$ by ∇ one obtains finally

$$\begin{aligned} V_{\mathrm{OPEP}} = \frac{f_\pi^2}{12\pi}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \Bigg[\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\left(\frac{\exp|-m_\pi r|}{r} - \frac{4\pi}{m_\pi^2}\delta(\mathbf{r})\right) \\ + S_{12}(\mathbf{r})\left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2}\right)\frac{\exp|-m_\pi r|}{r}\Bigg]. \end{aligned} \tag{3.43}$$

The exchange of a pseudoscalar meson leads to a spin-dependent central potential as well as a tensor part. As we have seen in (3.38), the exchange of a scalar meson, that is, $\rho(\mathbf{x}, \mathbf{r}) = g\delta(\mathbf{x} - \mathbf{r})$, leads to a central potential with no spin dependence. A more detailed derivation of the equations above can be found in many textbooks, for example, [BD64].

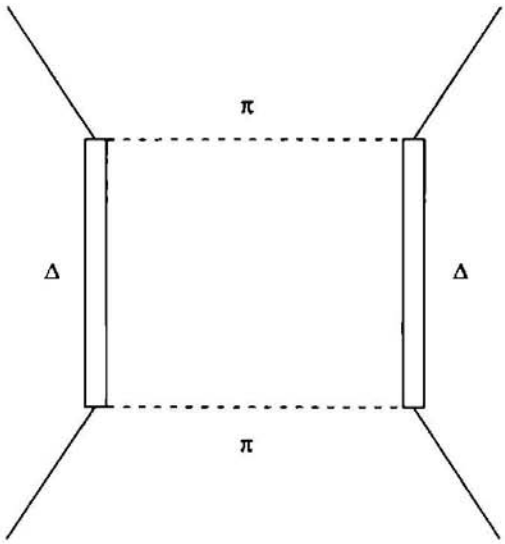

Figure 3.5 Nuclear force due to the exchange of two pions. The Δ-particle is a kind of polarized nucleon in the pion field generated by the other nucleon.

The OPEP potential describes well the nucleon-nucleon scattering for angular momenta $l \geq 6$. The high value of this limit shows that the OPEP potential describes the nuclear force in a reasonable way at great distances ($r \geq 2$ fm).

3.4.3 *Short range part of the NN interaction*

The short range part of the nucleon-nucleon potential represented in figure 3.2 is due to the exchange of three pions or more. The essential part of this process can be described by the effective exchange of a resonance of three pions, known as the ω-meson with spin 1 and $m_\omega = 783.8$ MeV. The ω exchange is important for two properties of the nuclear force: the repulsive part of the potential and the spin-orbit interaction.

Both properties also have analogies with the electromagnetic case. In the case of electromagnetism, the exchange of a photon also gives rise to the repulsive force between charges of the same sign. In the case of the nuclear force, due to the large ω-meson mass, the repulsive force is of short range. Starting from this argument we can also conclude that the strongly repulsive potential becomes a strongly attractive potential for a nucleon-antinucleon pair at short distances.

At intermediate distances the nucleon-nucleon potential is, as we have already emphasized, adequately described in terms of the exchange of two pions. Another way to describe this part of the potential is with the exchange of a single particle, the ρ-meson, of mass (768.1 ± 0.5) MeV/c^2. It is believed that this meson is built of two pions, thus the equivalence with the exchange of two pions follows.

The potentials derived from the hypothesis of π, ρ, and ω exchange consist of combinations of central, tensorial, and spin-orbit parts and terms of higher order. The

radial functions that accompany these terms have a total of up to 50 parameters, which are adjusted to the experimental data of the deuteron and to the nucleon-nucleon scattering.

3.4.4 Chiral symmetry

A meson is a complicated solution of the QCD quark and gluon fields that involves a real quark-antiquark pair. However, without ever being able to find this solution, we can identify basic features of the meson that result from the underlying QCD structure.

Let us concentrate on a small piece of the QCD Lagrangian density (1.59), that is, on the up-and-down quark components of the first term,

$$\mathcal{L}_\chi = -\bar{u}\gamma^\mu D_\mu u - \bar{d}\gamma^\mu D_\mu d = -\bar{q}\gamma^\mu D_\mu q.$$

The gluon fields and the color SU(3) matrices are not essential now, so we have hidden all that in the SU(3) covariant derivative: $D_\mu = \partial_\mu - igA^a_\mu t_a$. On the other hand, we have explicitly indicated the up-and-down quark fields, u and d, and moreover, we have combined the fields into the quark iso-spinor,

$$q = \begin{pmatrix} u \\ d \end{pmatrix}. \tag{3.44}$$

To be specific, q contains 24 components, i.e., two quarks, each in three colors, and each built as a four-component Dirac spinor. However, the Dirac and color structure is again not essential, so in the present section we may think about q as a two-component spinor. For a moment we have also disregarded the quark mass terms—we reinsert them to some degree below.

What is essential now are the symmetry properties of $\mathcal{L}_\chi$. This piece of the Lagrangian density looks like a scalar in the two-component field q, i.e., it is manifestly invariant with respect to unitary mixing of up-and-down quarks. We formalize this observation by introducing the isospin Pauli matrices, τ_1, τ_2, and τ_3, which are equal to the matrices defined in Appendix A (A.82), and we introduce unitary mixing of up-and-down quarks in the language of rotations in the abstract isospin space. This is exactly the same iso-space that we know very well from chapter 1, where the upper and lower components are the neutron and proton.

$\mathcal{L}_\chi$ is also invariant with respect to multiplying the quark fields by the γ_5 Dirac matrix (see Appendix D). This property results immediately from the commutation properties of the γ matrices (remember that $\bar{q} = q^\dagger\gamma_0$). So in fact we have altogether six symmetry generators of $\mathcal{L}_\chi$, namely,

$$\mathbf{I} = \tfrac{1}{2}\boldsymbol{\tau} \quad \text{and} \quad \mathbf{X} = \gamma_5\mathbf{t}\,, \tag{3.45}$$

where the boldface denote vectors in the isospace.

It is now easy to identify the symmetry group of $\mathcal{L}_\chi$. We introduce the left-handed $\mathbf{t}_L$ and right-handed $\mathbf{t}_R$ generators,

$$\mathbf{t}_L = \tfrac{1}{2}(1+\gamma_5)\mathbf{t} = \tfrac{1}{2}(\mathbf{t}+\mathbf{x}) \quad \text{and} \quad \mathbf{t}_R = \tfrac{1}{2}(1-\gamma_5)\mathbf{t} = \tfrac{1}{2}(\mathbf{t}-\mathbf{x}). \tag{3.46}$$

Since $(\gamma_5)^2 = 1$, they fulfill the following commutation relations:

$$[t_{Li}, t_{Lj}] = i\epsilon_{ijk}t_{Lk}, \quad [t_{Ri}, t_{Rj}] = i\epsilon_{ijk}t_{Rk}, \quad [t_{Li}, t_{Rj}] = 0; \tag{3.47}$$

that is, $\mathbf{t}_L$ generates the SU(2) group, $\mathbf{t}_R$ generates another SU(2) group, and since they commute with one another, the complete symmetry group is SU(2)×SU(2). We call this group *chiral*.

This result is in disagreement with experiment. On the one hand, particles appear in iso-multiplets. For example, there are two nucleons, a neutron and a proton, that can be considered as upper and lower components of an isospinor, and there are three pions, π_+, π_0, and π_-, that can be grouped into an isovector. So there is no doubt that there is an isospin SU(2) symmetry in nature, but, what about the second SU(2) group? In the Lorentz group, the γ_5 Dirac matrix changes the parity of the field, so if γ_5 were really a symmetry then particles should appear in pairs of species having opposite parities. This is not so in our world. Nucleons have positive intrinsic parity, and their negative-parity brothers or sisters are nowhere to be seen. Parity of pions is negative, and again, the positive-parity mirror particles do not exist with any being nearly of the same mass.

So nature tells us that the SU(2)×SU(2) symmetry of the QCD Lagrangian must be *dynamically broken*. It means that the Lagrangian has this symmetry, while the physical solutions do not. We have already learned that these physical solutions are very complicated, and we are unable to find them and check their symmetries. But we do not really need that—experiment tells us that chiral symmetry must be broken, and hence we can build theories that incorporate this feature on a higher level of description.

Let us now reinsert the quark-mass terms into the discussed piece of the Lagrangian:

$$\mathcal{L}'_\chi = -\bar{u}\gamma^\mu D_\mu u - \bar{d}\gamma^\mu D_\mu d - m_u\bar{u}u - m_d\bar{d}d\,. \tag{3.48}$$

Neither of the two mass terms, nor any linear combination thereof, is invariant with respect to the chiral group SU(2)×SU(2). For certain, had the quark masses been equal, the two combined mass terms would have constituted an isoscalar (an invariant with respect to the isospin group), but even then they would not be chiral scalars (invariants with respect to the chiral group). So, the nonzero quark masses break the chiral symmetry. What the values of these masses are needs to be taken from experiment, and indeed, the up-and-down quark masses are neither zero nor equal to one another. The chiral symmetry is therefore broken in two ways: explicitly, by the presence of a symmetry breaking term in the Lagrangian, and dynamically, as discussed above. Without going into details, we just mention that the nonzero mass of the π-mesons results from the nonzero quark masses; see [Wei99], chap. 19. For more quark flavors, when taken into account, the dimensionality of the chiral group increases; for example, when three quarks u, d, and s are considered the chiral group is SU(3)×SU(3).

3.4.5 Generalized boson exchange

So far we have used a purely classical approach for the nucleon-nucleon potential. The quantum mechanical treatment delivers the same result, if one relies on the lowest order perturbation theory in the static case. The starting point is a system of coupled nucleon and meson fields

$$H = H_N^0 + H_m^0 + H_{mN}, \tag{3.49}$$

whereby the free fields are described by H_N^0 and H_m^0. The meson-nucleon coupling H_{mN} depends on the meson type. The most important are the following three types (ψ denotes the nucleon field):

(1) *Scalar meson:* $\phi^{(s)}$ with coupling constants g_s.

$$H_{mN}^{(s)} = g_s \overline{\psi}\psi\phi^{(s)}, \tag{3.50}$$

(2) *Pseudoscalar mesons:* $\phi^{(ps)}$. Here one distinguishes pseudoscalar (ps) and pseudovector (pv) couplings.

$$H_{mN}^{(ps)} = ig_{ps}\overline{\psi}\gamma_5\psi\phi^{(ps)}, \tag{3.51}$$

$$H_{mN}^{(pv)} = i\frac{f_{ps}}{m_{ps}}\overline{\psi}\gamma_5\gamma^\mu\psi\, i\partial_\mu\phi^{(ps)} \tag{3.52}$$

with corresponding coupling constants g_{ps} and f_{ps}.

(3) *Vector mesons:* $\phi_\mu^{(V)}$. Here also one has two possibilities, vector coupling (γ^μ) with coupling constant g_V and tensor coupling with coupling constant f_V.

$$H_{mN}^{(V)} = g_V\overline{\psi}\gamma^\mu\psi\phi_\mu^{(V)} - \frac{f_V}{4M}\overline{\psi}\sigma^{\mu\nu}\psi\,(i\partial_\mu\phi_\nu^{(V)} - i\partial_\nu\phi_\mu^{(V)}) \tag{3.53}$$

with M = nucleon mass.

In these equations, the meson field is denoted by ϕ or ϕ_μ. Furthermore, the mesons carry isoscalar and isovector properties. In the latter case, the couplings $\overline{\psi}\cdots\psi\phi$ are to be replaced by $\overline{\psi}\cdots\boldsymbol{\tau}\psi\cdot\boldsymbol{\phi}$. For pseudoscalar mesons coupled to nucleons on the mass shell, the following relation holds:

$$\frac{f_{ps}}{m_{ps}} = \frac{g_{ps}}{2M}. \tag{3.54}$$

This relation is, however, no longer valid for off-shell situations. In particular, the antiparticle contributions to the ps-coupling are large, while they are strongly suppressed for the pv-coupling. At this point it is also important to recall that in processes for which the chiral symmetry is of importance, the pv-coupling is preferred, since in contrast to the ps-coupling the chiral symmetry holds for the pv-coupling.

In second order perturbation theory the *Feynman diagram* represented in figure 3.6 has an amplitude given by

$$\bar{u}_1(\mathbf{q}')\Gamma_1\bar{u}_1(\mathbf{q})G_m(q'-q)\,\bar{u}_2(-\mathbf{q}')\Gamma_2\bar{u}_2(-\mathbf{q}), \tag{3.55}$$

where $u(\mathbf{p})$ is the Dirac spinor of a nucleons, Γ_k is the vertex function arising from H_{mN}, and G_m is the free meson propagator. The vertex function Γ_k and propagator G_m depend

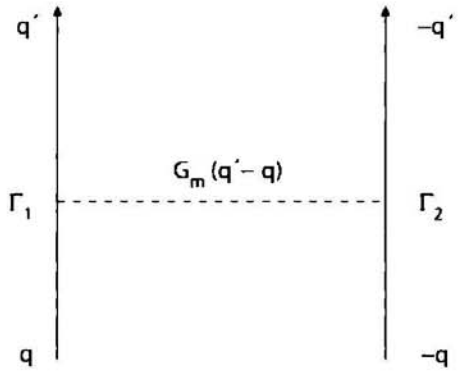

Figure 3.6 Feynman diagram for one-boson exchange.

on the meson type. For example, in the case of scalar and pseudoscalar mesons

$$G_m(q'-q) = \frac{1}{(q'-q)^2 - m^2}\,. \tag{3.56}$$

For vector mesons one must also take into account the spin structure. In the above relation $q \equiv (q_0, \mathbf{q})$ is a 4 vector.

The range of the potential described by a meson exchange is given by its Compton wavelength m^{-1}. An overview of the related mesons is given in table 3.5. One recognizes that the π-meson describes the long range of the interaction due to its low mass, while η, ρ, and ω are the responsible for the short range part of the interaction. Realistic potentials need, however, another meson of middle range (~500 MeV), to produce the necessary attraction at middle range (σ-meson). The existence of this meson is, however, disputed.

As an example of a realistic potential we discuss the *Bonn potential* given in coordinate space [Mac01]. The mesons included in this potential are shown together with their corresponding coupling constant in table 3.6. In each spin-isospin channel the potential

Table 3.5 Mesons related to the meson exchange problem with spin *J*, parity *P*, isospin *I*, *G*-parity *G*, and mass in MeV. *G*-parity is a combination of charge conjugation and a 180° rotation around the 2nd axis of isospin space (see, e.g., [Wei99]).

Meson	J^P	I^G	Mass
$\pi^{\pm}$	0^-	1^-	139.57
π^0	0^-	1^-	134.97
η	0^-	0^+	548.8
ρ	1^-	1^+	769
ω	1^-	0^-	769

Table 3.6 Mesons included in the Bonn potential, with their coupling constants and the cutoff parameters Λ ($n = 1$; see (3.58). Also used are $f_\rho/m_\rho = 6.1$ and $f_\omega/m_\omega = 0$.

Meson	Mass [MeV]	$g^2/4\pi$	Λ [GeV]
π	138.03	14.9	1.3
η	548.8	2	1.5
ρ	769	1.2	1.2
ω	782.6	25	1.4
δ	983	2.742	2.0
σ	550	8.77171	2.0

is written in the form

$$V = V_C + V_T S_{12} + V_{LS}\, \mathbf{L}\cdot\mathbf{S}. \tag{3.57}$$

Vertex functions were also introduced in these potentials to account for the finite size of the nucleons and mesons. One often chooses the simple analytical form

$$F(q^2) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}\right)^{n/2} \qquad (n = 1, 2, \ldots), \tag{3.58}$$

with a suitable choice of cutoff parameters that are also shown in table 3.6. The high momentum components are suppressed; in particular, the δ-function in the scalar part of the potential is eliminated. Other functional forms are also used, for example, Gaussian functions. In coordinate space, the introduction of form-factor functions is viewed as a weakening of the potential at short distances. In principle, this is a purely heuristic procedure based on our ignorance of the interaction at short distances. It parametrizes this part of the interaction in the simplest way.

With regard to the role of the different mesons, the following qualitative features arise:

(a) The long range part ($r > 2$ fm) is described exclusively through the pion. Also, modern phenomenological potentials account for this fact.

(b) The middle range part ($1 < r < 2$ fm) is dominated by attraction. That is described through two-pion exchange (TPE).

(c) The short range part is dominated by ρ and ω exchange, responsible for a strong repulsion.

(d) Finally, the very short range is described purely phenomenologically, either through a sharp cutoff radius ("hard core"), or in a soft form ("soft core").

3.4.6 *Beyond boson exchange*

One of the main problems of the OBE potentials is justification of the σ-exchange that is needed for description of the attraction at middle range. Within the set of physical mesons,

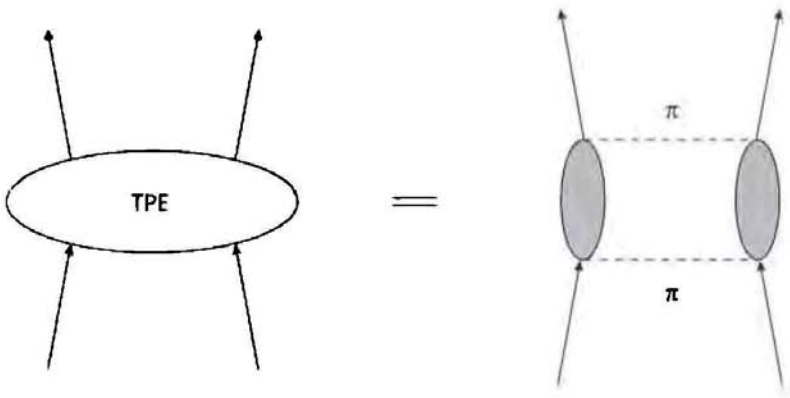

Figure 3.7 Two-pion exchange diagram.

there is no meson with the demanded characteristics. Investigations have shown that the underlying mechanism is a correlated two-pion exchange (TPE). Two methods are often used to attack this problem.

(1) Dispersion relation method. This is based on the assumption that the TPE contribution, shown diagrammatically in figure 3.7, can be cut in the middle and separated into two pion lines representing two disjunct pion-nucleon off-shell scatterings. The off-shell pion-nucleon scattering is separated into a sum of pure pion-nucleon scattering without $\pi\pi$ scattering and an additional contribution with $\pi\pi$ scattering. This is shown schematically in figure 3.8. The two-pion exchange amplitude is then written as a dispersion relation integral

$$T_{2\pi}(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} dt' \, \frac{\rho_{2\pi}(t')}{t' - t}. \tag{3.59}$$

where $t = (p - p')^2$ is a *Mandelstan variable.* In this integral $\rho_{2\pi}(t)$ is called the *spectral function.* It characterizes both the strength and the range of the interaction and it is obtained by the "crossing" symmetry from the process $N\overline{N} \to 2\pi$.

One can show that with the dispersion relation (3.59) one can obtain $\rho_{2\pi}$ from $\pi\pi$ and πN scattering amplitudes [ChR79]. The contribution to the NN potential is then given by

$$\frac{1}{r}\int_{4m_\pi^2}^{\infty} dt' \, \rho_{2\pi}(t') \, e^{-\sqrt{t}r}. \tag{3.60}$$

The *Paris potential* is constructed with this method. The explicit form is further parametrized in terms of a sum of Gaussians [Lac80].

(2) Field theory approach. Here, as in the OBEP, one uses an explicit field theory model to obtain the TPE amplitude. The lowest order diagrams are represented in figure 3.9. Relevant for a realistic description are the couplings to resonances, most importantly the $\Delta(1232)$ resonance, together with pure $\pi\pi$ exchange. The nucleon resonances are herewith treated as elementary particles. One gets in this manner a detailed microscopic description of the $\pi\pi$ exchange. In contrast to the simple OBE potentials, the correlated $\pi\pi$ exchange is described here in terms of realistic ρ and σ with wide mass distribution.

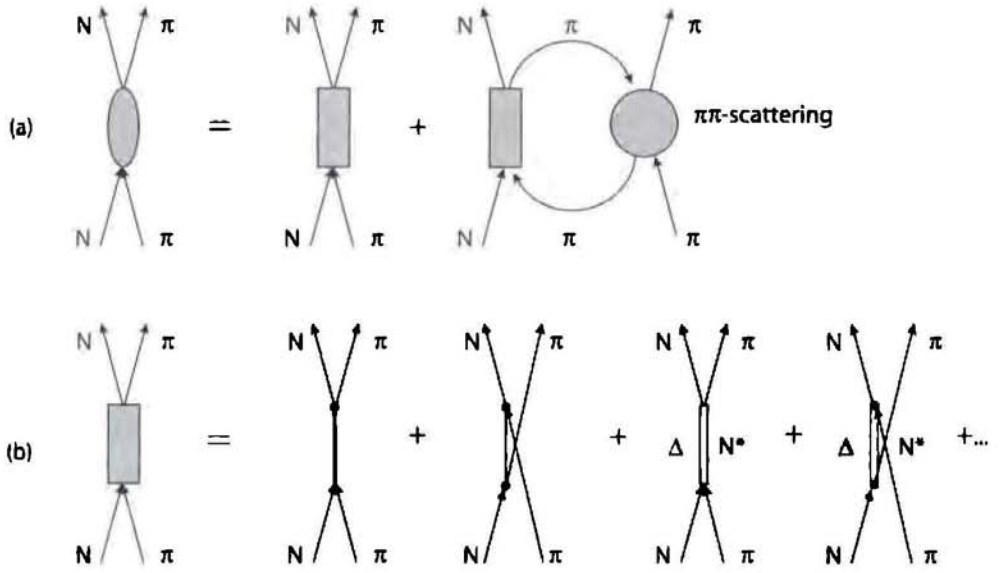

Figure 3.8 Off-shell pion-nucleon scattering separated into a pure πN part and an additional part with $\pi\pi$ scattering [ChR79].

With regard to the meaning of 2π exchange one can raise the question how far in higher order diagrams one has to go and in which manner one can undertake a systematic development. That is represented schematically in figure 3.10, where the arrows indicate which diagrams must be considered simultaneously in order to reach convergence.

Parallel to these refinements that follow in the frame of conventional nuclear physics, the question arises in the development of QCD as a fundamental theory of the strong interaction as to how the NN interaction can be described in this framework. The basic difficulty to answering this question, given that the structure of the hadrons is already complicated by the nuclear many-body environment, lies in the so-called nonperturbative area of QCD, dominated by the phenomenon of *confinement* and described by nonlinear equations of motion.

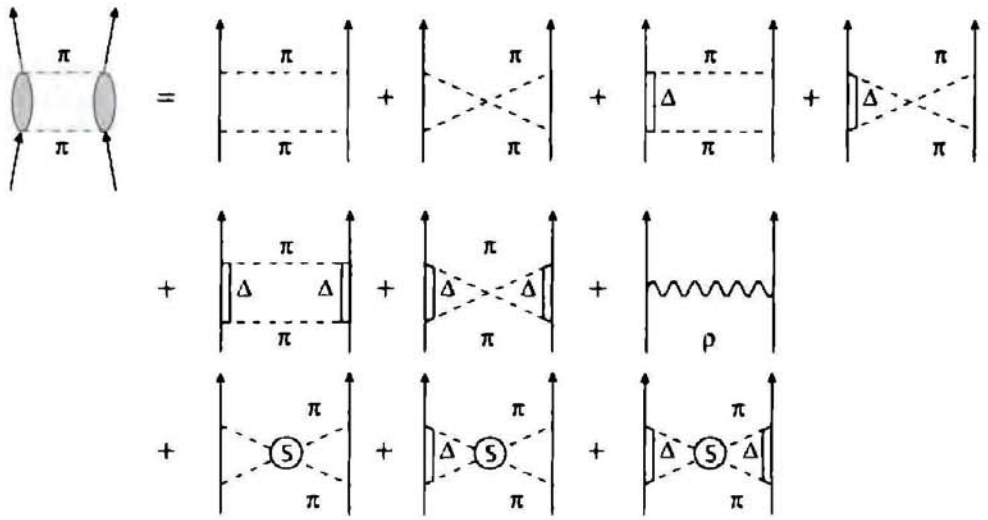

Figure 3.9 Description of two-pion exchange in the field theory approach.

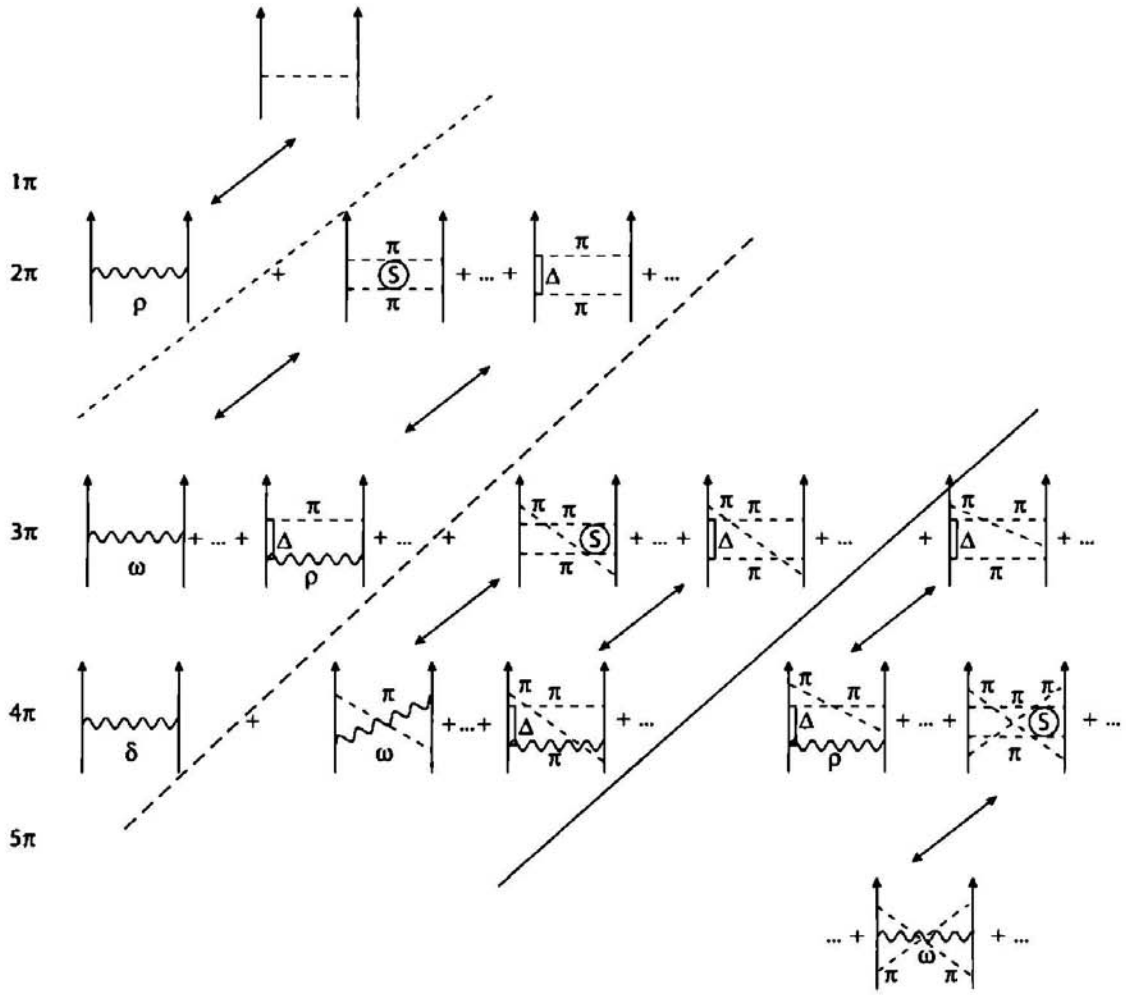

Figure 3.10 Systematic development of the field theory approach [Mac89].

For this reason hybrid models have been developed, in which the internal hadron structure is described with effective quark models. The interaction is described in these models in the long range part through conventional meson exchange. However, the mesons are now coupled directly to the quarks, so the extended structure of the meson-nucleon vertices

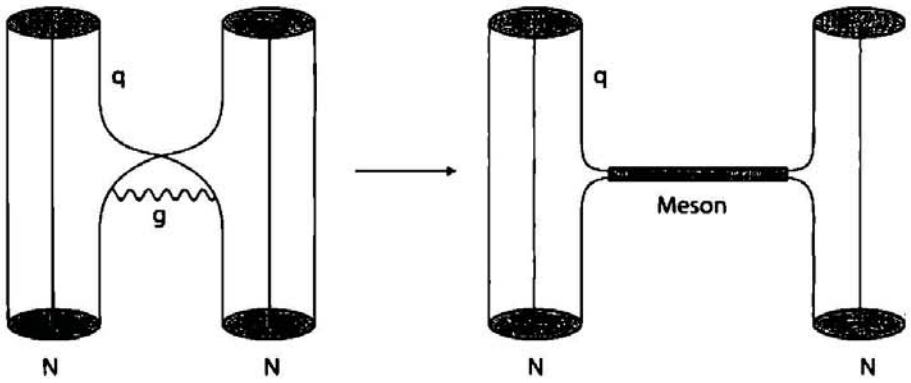

Figure 3.11 Quark-gluon exchange between two nucleons at small distances (left) and effective meson exchange at larger distances (right).

is obtained automatically by the quark wavefunctions. Only for very short distances does the interaction become dominated by explicit quark-gluon exchange (see figure 3.11).

3.5 Effective Field Theories

As we have discussed earlier, the Argonne v18 interaction uses the OPEP potential at large distances and the phenomenological interaction at intermediate and small distances. One can also follow the standard approach of quantum field theory and model the second piece by the exchange effects for heavier mesons. Larger meson masses mean shorter distances of the interaction, so we can understand why, by adding more mesons and using the corresponding Yukawa interactions, we can parametrize the NN force equally well.

Although this way of proceeding works very well in practice, it creates two conceptual problems. First, one has to include the scalar-isoscalar meson called σ, which has the quantum numbers of a pair of pions. It fulfills the role of an exchange of a pair of pions. However, such a meson does not exist in nature as a free particle, nor is its mass (which has to be used in the corresponding Yukawa term) close to the doubled pion mass. The exchange of such a virtual particle simply corresponds to higher order effects in the exchange of pions, which is a perfectly legitimate procedure, but it departs from the idea that real, physical particles mediate the NN interaction. Second, two other heavy mesons have to be included, the vector isovector meson ρ and the vector isoscalar meson ω. They are physical particles, with rest masses of about 800 MeV, and the corresponding ranges of the Yukawa potentials are very small, of the order of 0.25 fm. These small ranges allow modeling the NN interaction at very short distances, but at these distances nucleons really start to touch and overlap. Therefore, it is rather unphysical to think that nucleons can still interact as unchanged objects by exchanging physical particles. Within the image of the strong color polarization taking place at such a small distances, one would rather think that the internal quark-gluon structure of nucleons becomes strongly affected, which creates strong repulsion effects, predominantly through the Pauli blocking of overlapping quark states.

At present, we are probably not at all able to tell what happens with the nucleons when they are so near to one another. However, we do not really need such complete knowledge when describing low energy NN scattering and structure of nuclei. All we need is some kind of parametrization of the short range, high energy effects when we look at their influence on the long range, low energy observables. Such separation of scales is at the heart of *effective field theory* (EFT).

One can apply similar ideas to almost all physical systems, where our knowledge of the detailed structure is neither possible nor useful. The simplest example is the effect of the electromagnetic charge and current distributions inside a small object, when we shine at it an electromagnetic wave of a much longer length (the long wave length limit). It is well known that all we need are a few numbers—low multiplicity electric and magnetic moments. Of course, the best would be to be able to calculate these moments from the exact charge and current distributions, but once we know these numbers, we know everything.

On the other hand, if the internal structure is not known, we can fit these numbers to the measured long wave scattering and thus obtain the complete information needed to describe such a scattering process.

Examples of other such situations are plenty in physics. Interested students are invited to go through the very good introductory lecture notes by Lepage [Lep97], where nice instructive examples are presented in the framework of ordinary quantum mechanics. In particular, it is shown how a short range perturbation of the ordinary Coulomb potential influences the hydrogen atomic wavefunctions, and how such a perturbation (no matter what its physical origin) can be parametrized by a zero range, delta-like potential.

3.6 Exercises

1. Suppose that the interaction potential between the neutron and the proton is exponential, of the form $V = V_0 e^{-r/2r_n}$, where V_0 and r_n are, respectively, the depth and the range of the nuclear potential. a) Write the Schrödinger equation (in the center of mass system) for the ground state of the deuteron of angular momentum $l = 0$. b) Use the definition $x = e^{-2r/r_n}$ and $\psi(r) = u(r)/r$. Show that the Schrödinger equation has a Bessel function as a solution. Write the general solution of this equation. c) Applying the boundary conditions (ψ finite for $r = 0$ and $r = \infty$), determine the relationship between V_0 and r_n.

2. For a system of two nucleons show that $L + S + T$ should be odd, where L, S, and T are, respectively, the quantum numbers of orbital momentum, spin, and isospin of the system.

3. The deuteron has spin 1. What are the possible states of total spin and of total angular momentum of two deuterons in a state with orbital angular momentum L?

4. A particle with spin 1 moves in a central potential of the form

$$V(r) = V_1(r) + \mathbf{S} \cdot \mathbf{L} V_2(r) + (\mathbf{S} \cdot \mathbf{L})^2 V_3(r).$$

What are the values of $V(r)$ in the states with $J = L + 1$, L, and $L - 1$?

5. Suppose that the meson π^- (spin 0 and negative parity) is captured from the orbit P in a pionic atom, giving rise to the reaction

$$\pi^- + d \to 2n.$$

Show that the two neutrons should be in a singlet state.

6. Consider the operator S_{12} defined in (3.9). Show that, for the spin singlet and triplet state of the two particles, the following relations are valid:

$$S_{12}\chi_{\text{singlet}} = 0, \quad (S_{12} - 2)(S_{12} + 4)\chi_{\text{triplet}} = 0.$$

7. Let $\mathbf{s}_1$ and $\mathbf{s}_2$ be the spin operators of two particles and $\mathbf{r}$ the radius vector that connects them. Show that any positive integer power of the operators

$$\mathbf{s}_1 \cdot \mathbf{s}_2 \quad \text{and} \quad \frac{3(\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r})}{r^2} - (\mathbf{s}_1 \cdot \mathbf{s}_2)$$

can be written as a linear combination of these operators and the unit matrix.

8. Prove the relations

$$\mathbf{s}_1 \times \mathbf{s}_2 = \frac{2i}{h}(\mathbf{s}_1 \cdot \mathbf{s}_2)\mathbf{s}_1 - \frac{ih}{2}\mathbf{s}_2,$$

$$(\mathbf{s}_1 \times \mathbf{r}) \cdot (\mathbf{s}_2 \times \mathbf{r}) = R^2(\mathbf{s}_1 \cdot \mathbf{s}_2) - (\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r}).$$

9. Show that the tensorial force S_{12} has a zero angular average; that is, show that $\int S_{12} d\Omega = 0$.

10. Find the functional form in coordinate space of a potential expressed in momentum space by equation (3.58).

11. Consider, in addition to nuclear np forces, the interaction of the neutron magnetic moment with the Coulomb field of the proton.

(a) Show that, for nonrelativistic relative np motion, this interaction leads to a new Hamiltonian term with the structure

$$H' = V(r)(\mathbf{l} \cdot \mathbf{s}_n). \tag{3.61}$$

where $\mathbf{l}$ and $\mathbf{s}_n$ are the operators of the relative orbital momentum and of the neutron spin, respectively. Find the coordinate dependence of $V(r)$.

(b) Find the constants of motion in the *np* system in the presence of the additional interaction (3.61).

(c) Construct the matrix of interaction (3.61) in the basis of the unperturbed n-p wavefunctions with given values of l, total spin S, and total angular momentum J. Estimate the shift of the deuteron binding energy due to this interaction.

(d) Write down the Schrödinger equations for the radial wave functions outside the range of nuclear forces. Are there any new effects expected in the np scattering?