Physics 203

H. W. Assignment 2

1.) Show that the deuteron binding energy ($E_b = 2.22$ MeV) and $n-p$ triplet scattering length ($a_t = +5.4$ fm) can be reproduced by a simple square well [$V(r) = -V_0, r \leq r_0, V(r) = 0, r > r_0$] by determining the parameters V_0 and r_0. Use these parameters to estimate the effective range in $n-p$ triplet scattering, where the effective range (r_e) is defined from the expansion:

$$k\cot(\delta_0) = -1/a_t + r_e k^2/2 + \ldots$$

2.) In molecular hydrogen two types of states are discussed: ortho- and para- hydrogen. The ortho is the state that is symmetric with respect to spin exchange, while para is antisymmetric. The same nomenclature is applied to molecular deuterium, with ortho again being the spin symmetric state.

(a) Use the fact that protons are spin 1/2 and deuterons are spin 1 to explain why (equations are OK) parahydrogen is the ground state of the hydrogen molecule while orthodeuterium is the ground state of the deuterium molecule.

(b) What are the allowed values of the orbital angular momentum quantum number L for ortho- vs para-deuterium?

Note that ortho-deuterium plays a big role in the production of so-called "Ultra-Cold Neutrons" in neutron moderators made from solid deuterium.

3.) We want to make an estimate of the D-state admixture in the ground state of the deuteron. We will assume a wave function of the form

$$\psi = a\phi_{1s} \ + \ b\phi_{1d}$$

with $b << a$ and a, b are real. Assume that the ϕ's are given by 3-d harmonic oscillator wave functions, with radial wave functions given by

$$R_{1s}(r) = 2\pi^{-1/4}\alpha^{3/4}\exp(-\alpha r^2/2)$$
$$R_{1d}(r) = 4(15)^{-1/2}\pi^{-1/4}\alpha^{7/4}r^2\exp(-\alpha r^2/2)$$

and angular wave functions given by the appropriate Y_{lm}'s. First calculate the rms radius for the 1s state [$(< \phi_{1s}|r^2|\phi_{1s} >)^{1/2}$] assuming the radius is not sensitive to the D-state admixture. Then fix the oscillator parameter α to give the measured deuteron radius ($r = 1.96$ fm). Now calculate the quadrupole moment $< \psi|\hat{Q}|\psi >$, and determine the D-state percentage by adjusting b/a to give the measured deuteron quadrupole moment ($Q = 0.286$ efm^2). Recall that the quadrupole moment operator is given by

$$\hat{Q} = e(3z^2 - r^2) = (16\pi/5)^{1/2} e r^2 Y_{20}$$

4.) Consider simple $n-p$ scattering at low energies (i.e. s-wave scattering only). For unpolarized beam and target, there are four possible initial (and final) spin states for the two nucleons:

$$|+\tfrac{1}{2}+\tfrac{1}{2}\rangle\ ,\ |+\tfrac{1}{2}-\tfrac{1}{2}\rangle\ ,\ |-\tfrac{1}{2}+\tfrac{1}{2}\rangle\ ,\ |-\tfrac{1}{2}-\tfrac{1}{2}\rangle\ ,$$

where the two numbers in each state are the m quantum number of spin for the neutron and proton ($|m_p\ m_n\rangle$. Using the measured scattering lengths of the $n-p$ system ($a_s=-23.7$ fm, $a_t=+5.4$ fm), determine the probability of spin-flip occurring in the interaction.

5.) Bertulani problems 2-4 & 2-12.