SP3

You can use the following relation:

$$\nabla(\boldsymbol{\sigma}\cdot\nabla f) = \hat{r}(\boldsymbol{\sigma}\cdot\hat{r})\left[\frac{\partial^2 f}{\partial r^2} - \frac{1}{r}\frac{\partial f}{\partial r}\right] + \boldsymbol{\sigma}\frac{1}{r}\frac{\partial f}{\partial r}$$

where $\hat{r}$ is an unit vector and $\nabla_2 = -\nabla_1 = \nabla$, to solve the following problem:

At distances sufficiently large that overlap between their densities may be ignored, the interaction between two nucleons may be shown to be similar to that between two point dipoles,

$$V(r) \sim (\boldsymbol{\sigma}_1\cdot\nabla_1)(\boldsymbol{\sigma}_2\cdot\nabla_2)f(r)$$

Under the assumption of one-pion exchange, we may take the radial dependence to have the form

$$f(r) = \frac{e^{-r/r_0}}{r}$$

where

$$r_0 = \frac{\hbar c}{m_\pi c^2}$$

is the range. The strength of the potential may be related to the pion-nucleon coupling constant g ($g^2/\hbar c \simeq 0.081 \pm 0.002$). Except for isospin dependence, which we shall ignore here for simplicity, the potential may be written as

$$V(r) = -g^2 r_0^2 (\boldsymbol{\sigma}_1\cdot\nabla_1)(\boldsymbol{\sigma}_2\cdot\nabla_2)\frac{e^{-r/r_0}}{r}$$

Use the result of Problem 3-12 above to show that $V(r)$ can be expressed in terms of the tensor operator S_{12} given in Eq. (3-38),

$$V(r) = \frac{g^2}{3}\left\{\left[\left(1 + \frac{3r_0}{r} + \frac{3r_0^2}{r^2}\right)S_{12} + \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]\frac{e^{-r/r_0}}{r} - 4\pi r_0^2\delta(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right\}$$

where $r = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$.