PHYSICS 2c – Quiz 2

This exam covers all of the assigned readings in KK, through HW 4 and chapter 4 in KK, with a focus on the second two weeks of class.

This is an OPEN BOOK exam with the following limitations: You may consult only your textbook (Kittel+Kroemer), notes that you have taken in recitation or lecture, online lecture notes, and HW solns (including your own). No other references are allowed. You may use a calculator and symbolic manipulation programs (eg. Mathematica) only for doing arithmetic, evaluating integrals or simplifying algebra, although they are not required.

In your studies, do not look at problems or exams from previous years of Phys 2 or 12.

The time limit is 2.0 HOURS, in one continuous sitting. A 15 minute break beyond the 2 hours is permitted during this period, as long as you are not working on the exam during this time. No credit for overtime work.

This Quiz is DUE Tuesday, May 3 at 4:00PM and should be turned into the HW lock-box. Late Quizzes will not be accepted for credit except by prior arrangement with the instructor.

There are 3 problems on pages 2–3 for a total of 11 points.

To get partial credit, show as much work as you can!

Please use a blue book or stapled sheets of paper

DON’T FORGET TO WRITE YOUR NAME clearly on the front of your exam.
Problem 1 3-state System

Consider a system of $N$ atoms in thermal equilibrium with a reservoir at temperature $T > 0$. Treat the atoms as localized and distinguishable objects with internal energy states but no interactions with the other atoms. Each atom has the following non-degenerate energy spectrum:

(i) A ground state with $\epsilon_0 = 0$; (ii) a first excited state with $\epsilon_1 = \Delta$; (iii) a second excited state with $\epsilon_2 = 2\Delta$.

(a) (1 point) Evaluate the one-particle partition function, $Z_1$.

(b) (0.5 point) What is the $N$-particle partition function, $Z_N$?

(c) (0.5 point) What is the $N$-particle Helmholtz free energy, $F(T)$?

(d) (1 point) What is the entropy of the $N$-particle system?

(e) (0.5 point) Show that this system satisfies the third law of thermodynamics.

(f) (1 point) What is the average energy of the $N$-particle system $U(T)$?

(g) (0.5 point) What is the average energy in the limits: $T \to 0, T \to \infty$?

Problem 2 - Photon Gas

(2 points) For an isothermal (meaning at constant temperature) expansion of a photon gas in a cavity (a black body) from a volume $V$ to a volume $2V$, do the following quantities increase, decrease, or remain constant? Answer separately for each one, and be sure to explain your reasoning.

Entropy, $\sigma$

Energy density, $u = U/V$

Total number of photons, $N$

Free energy, $F$
Problem 3 - Hot Spheres

Consider two concentric spheres: \( S_1 \) with radius \( r_1 \) and \( S_2 \) with radius \( r_2 \) and \( r_2 > r_1 \) as shown below. \( S_1 \) is maintained at temperature \( T_1 \) and \( S_2 \) at \( T_2 \). Assuming that both spheres behave as perfect black-bodies, if \( T_2 > T_1 \) then there is a net energy transferred from \( S_2 \) to \( S_1 \). Clearly \( S_2 \) is emitting energy at a rate \( \sigma T_2^4 \times A_2 \), where \( A_2 \) is the area of \( S_2 \). But all of this energy does not reach \( S_1 \) since it is emitted uniformly from the surface so that much of it strikes a different part of the surface of \( S_2 \) and completely misses \( S_1 \). In constrast all of the energy that leaves \( S_1 \) strikes somewhere on \( S_2 \). The issue of what happens to this net energy goes is the subject of part (b), but first ...

\[
\frac{\Delta E}{\Delta t} = \sigma (T_2^4 - T_1^4) \times 4\pi r_1^2
\]

Let’s use the above to do a real world calculation.

(a) (1 point) Show that the net energy transfer rate from \( S_2 \) to \( S_1 \) is given by

(b) (1 point) We will now fill the \( S_1 \) sphere with liquid helium ((LHe) at a temperature of 4K and surround it with a vacuum chamber \( S_2 \) at room temperature so that radiation is the only possible source of energy transfer. Assuming that \( r_1 = 0.1 \) m, \( r_2 = 1.0 \) m, \( T_1 = 4K \), \( T_2 = 292K \). What is the energy transfer rate from \( S_2 \) to \( S_1 \) in joules/s = watts?

(c) (1 point) LHe has a ”latent heat of vaporization” of \( 2.08 \times 10^4 \) j/kg. This means that 20,800 joules of energy are needed to boil 1 kg of LHe. Using the density of LHe of 125 kg/m\(^3\), how long does it take for the helium to boil completely away through a small tube to the outside (not shown)?

(d) (0.5 point) We would like the LHe to last longer, so we surround \( S_1 \) with another sphere with intermediate radius \( r_3 = 0.5m \), again with vacuum between this new sphere and \( S_1 \). But now we cool this sphere to liquid nitrogen (LN) temperatures (\( T_3 = 77K \)). How long does the LHe last in this case?

(e) (0.5 point) If, in part (b), we had filled \( S_1 \) with LN (latent heat = 198,000 j/kg, density = 809 kg/m\(^3\)), how long would it take for all of the LN to boil away?