2.1 POSTULATES OF STATISTICAL MECHANICS

In constructing our theories of gravitation, we should be wary about accepting too glibly many of the prejudices of the present scientific thinking. In the last lecture we saw how there was something unsatisfactory in the way that probabilities appear in our interpretation of the universe. If we truly think that the universe is described by a grand wavefunction without an outside observer, nothing can ever become a probability, since no measurement is ever made! This many physicists hold in spite of our experimental evidence in dealing with subregions of the universe, which are for our purposes very accurately described by wavefunctions which represent amplitudes for the probabilities of results of measurements.

In a similar fashion, there are difficulties with the usual simple textbook description of statistical mechanics; although not closely related to the theory of gravitation it is related to cosmological questions which we shall discuss later. Often one postulates that a priori, all states are equally probable. This is not true in the world as we see it. This world is not correctly described by the physics which assumes this postulate. There are in the world people—not physicists—such as geologists, astronomers, historians, biologists, who are willing to state high odds that when we look into an as yet unobserved region of the universe, we shall find certain
organization which is not predictable by the physics we profess to believe. In our experience as observers, we find that if we look inside a book which says “Napoleon” on the cover, the odds are indeed very great that there will be something about Napoleon on the inside. We certainly do not expect to find a system in thermodynamic equilibrium as we open the pages. But the physicists have not found a way of incorporating such odds into unobserved regions of the universe. The present physics would never predict as well as a geologist the odds that when we look inside certain rocks, we will find a fossil.

In the same way, astronomical historians and astronomers find that everywhere in the universe that we look we see stars, which are hotter on the inside and cooler on the outside, systems which are indeed very far from thermodynamic equilibrium.

We can get an idea of how unlikely this situation is, from the point of view of the usual prejudices of thermodynamics, by considering simple orderings. Consider a box as the universe, in which there are two kinds of particles, white and black. Suppose that in a certain region of the universe, such as a small corner of a box, we see that all white particles are separated from the black particles along the diagonal (Figure 2.1). The a priori probability that this should be is very, very small, since we must, with our present prejudices, ascribe it to a statistical fluctuation which is rather unlikely. What do we predict for the rest of the universe? The prediction is, that as we look into the next region, we should most likely find that the next region has a much less ordered distribution of the white and black particles. But in fact we do not find this, but each new region that we see, we see the same order as before. If I get into my car and drive to the mountains I have never seen previously, I find trees that look just like the trees that I know. With the extremely large numbers of particles that are involved, the probabilities of such observations in terms of the usual statistical mechanics are fantastically small.

The best explanation of a fluctuation that one observes is that only that much has fluctuated and that the rest of the stuff is at random. If all states were equally likely a priori, and if one found a piece of the world
lopsided, then the rest of the world should be uniformly mixed because it would be less of a fluctuation. It may be objected that the events and structures in the world are correlated; they all have the same past! But that is a different theory than the one underlying the statistical mechanical description of the universe. There is this opposite theory, which is that in the past the world was more organized than now, and that the most likely state is not that of equilibrium, but some special state which is dynamically evolving. This is the common sense assumption which all historians, paleontologists, and others adopt.

Probability arguments must be used as a test for the theory, and may be employed as follows. Let us suppose that on \textit{a priori} grounds we wish to assign very, very low odds to the hypothesis that the universe is not to be described as an elaborate fluctuation from the complete chaos characterizing thermodynamic equilibrium; for example, let us suppose that the \textit{a priori} probability for the idea that all states are equally likely is \(1 - 10^{-100}\). Then let us describe a number of ordered states according to some scheme; for example, suppose that we list all states whose ordering can be specified by less than a million words. Now we assign the remaining \textit{a priori} probability of \(10^{-100}\) to the hypothesis that the universe is evolving from one of these special ordered conditions in the past. In other words, we adopt the prejudice that all states are equally likely, but wish to allow the possibility that observational tests might invalidate the equilibrium hypothesis.

Now we begin to make observations on the world about us, and we observe states with a describable ordering. Each one of us this morning saw that the ground was below and the air was above, but one such observation is enough to increase greatly the odds for the ordered states in the \textit{a posteriori} judgment on the probability of the initial situation. And as we make more and more observations, this increase eventually overtakes even the \(10^{-100}\), in a way which may be computed according to a theorem: if the \textit{a priori} probability of situation \(A\) is \(P_a\) and the \textit{a priori} probability of situation \(B\) is \(P_b\), and if an observation is made which is more likely if \(A\) holds and less likely if \(B\) holds, the \textit{a posteriori} probability of \(A\) increases by the ratio by which the result of the measurement is more probable if \(A\) holds.

If one makes an observation of a corner of the universe, anything macroscopic, one finds that it is far removed from equilibrium. The odds that this should be a fluctuation are extremely small; it requires only a single observation of macroscopic order to reduce the probability to \(10^{-2000}\), for which only 5000 molecules have to be ordered. Thus, it is perfectly obvious that only special states could possibly give rise to the immense degree of order we see in the world.

How then does thermodynamics work, if its postulates are misleading? The trick is that we have always arranged things so that we do not do experiments on things as we find them, but only after we have thrown out
precisely all those situations which would lead to undesirable orderings. If we are to make measurements on gases, which are initially put into a metal can, we are careful to “wait until thermodynamic equilibrium has set in,” (how often have we heard that phrase!) and we throw away all those situations in which something happens to the apparatus, that the electricity goes off because a fuse is blown, or that someone hits the can with a hammer. We never do experiments on the universe as we find it, but rather we control things to prepare rather carefully the systems on which we do the experimenting.

A more satisfactory way of presenting the postulates of statistical mechanics might go as follows. Suppose that we do know all details of a (classical) system, such as a mass of gas with infinite precision; that is, we know the positions and velocities of all the particles at some instant of time \( t = 0 \). We can then (disregarding the actual difficulties in practice) calculate exactly if we know the laws of nature exactly, and find out the behavior and state of all other particles at any time in the future. But now suppose that there is some small uncertainty in our measurements, or in our knowledge of any one fact that enters into the calculation, the position, the velocity, of any one particle, or a small uncertainty in the exactness with which we know the interactions of the particles. It does not at all matter (excepting counterexamples designed by mathematicians) where the uncertainty is. If such an uncertainty exists, we shall have to describe the final state by averaging over the uncertainty, and if a sufficiently long time elapses, which will be shorter the larger the uncertainty and the larger the system, the predictions of measurements will be very close to those given by the canonical theory of thermodynamic equilibrium.

If for example we plot the velocity of molecule number 6 at the time \( t = 30 \) min. as a function of any other starting variable in the system, such as the initial position or velocity of particle number 133, we shall find an extremely complicated curve of very, very fine detail, which should average to the “equilibrium” results as soon as we average over the initial finite uncertainty of the variable in question. In other words, the distribution of final values in the range considered should be much like the “equilibrium” distribution (Figure 2.2).

A physically satisfactory discussion of thermodynamics and statistical mechanics can only be achieved if it is recognized that the problem is to define conditions in a system in which various things happen at very different rates. Only if the rates are sufficiently distinct is thermodynamics of use. Thus the theory of thermodynamics must distinguish between slow and fast processes. When we are talking about thermodynamic equilibrium for our mass of gas, we do not wait for an infinite time, but for a time very long compared to a certain class of interaction (for example molecular collisions) which is producing the type of equilibrium we are considering. In studying oxygen in a metal can, we do not wait so long
that the walls of the can have oxidized, or that the metal has evaporated into space, as it eventually should do, since it has a finite vapor pressure, nor do we consider all the nuclear reactions that once in a very great while do (according to our theory) occur for the colliding molecules.

We must be careful to interpret the results of our theories when they are treated with full mathematical rigor. We do not have the physical rigor sufficiently well defined. If there is something very slightly wrong in our definition of the theories, then the full mathematical rigor may convert these errors into ridiculous conclusions.

The question is how, in quantum mechanics, to describe the idea that the state of the universe in the past was something special. The obvious way is to say that the wave function of the world (if such exists) was a certain $\psi_0$ at $t = -\infty$ (age of the universe). But that means the wave function $\psi$ at present tells us not only about our world but equally of all the other possible universes that could have evolved from this same beginning. This
is the cat paradox on a large scale. Equally represented is “our world” plus all other dead cats whose death was a quantum controlled accident. From this “our world” can be got by “reduction of the wave packet.” What is the mechanism of this reduction? You must either suppose that observing creatures do something not described by quantum mechanics (i.e., Schrödinger equation) or that all possible worlds which could have evolved from the past are equally “real.” This is not to say that either choice is “bad,” but only to point out that I believe that present quantum mechanics implies one or the other idea.

2.2 DIFFICULTIES OF SPECULATIVE THEORIES

In constructing a new theory, we shall be careful to insist that they should be precise theories, giving a description from which definite conclusions can be drawn. We do not want to proceed in a fashion that would allow us to change the details of the theory at every place that we find it in conflict with experiment, or with our initial postulates. Any vague theory that is not completely absurd can be patched up by more vague talk at every point that brings up inconsistencies—and if we begin to believe in the talk rather than in the evidence we will be in a sorry state. Something of this kind has happened with the Unified Field Theories. For example, it may be that one such theory said that there is a tensor \( J_{\mu\nu} \) which “is associated” with the electromagnetic tensor. But what does this “associated” mean? If we set the thing equal, the theory predicts wrong effects. But if we don’t specify “associated,” we don’t know what has been said. And talk that this “association” is meant to “suggest” some new relation leads to nowhere. The wrong predictions are ascribed to the wrong “suggestions” each time, rather than to the wrong theory, and people keep thinking of adding a new piece of some antisymmetric tensor which would somehow fix things up. This speculative talk is no more to be believed than the talk of numerologists who find accidental relations between certain magnitudes, which must be continuously modified as these magnitudes are measured with more precision, first relating units, and then smaller and smaller fractions of these units to keep up with the smaller and smaller uncertainties in the measured values.

In this connection I would like to relate an anecdote, something from a conversation after a cocktail party in Paris some years ago. There was a time at which all the ladies mysteriously disappeared, and I was left facing a famous professor, solemnly seated in an armchair, surrounded by his students. He asked, “Tell me, Professor Feynman, how sure are you that the photon has no rest mass?” I answered “Well, it depends on the mass; evidently if the mass is infinitesimally small, so that it would have no effect whatsoever, I could not disprove its existence, but I would be glad to discuss the possibility that the mass is not of a certain definite
We know that the most probable values of the energies $E_a$ are their mean values $E_a'$. This means that the function $S(E_1, E_2, \ldots)$ must have its maximum possible value (for a given value of the sum $\sum E_a = E_0$) when $E_a = E_a'$. But the $E_a$ are just the values of the energies of the subsystems which correspond to complete statistical equilibrium of the system. Thus we reach the important conclusion that the entropy of a closed system in a state of complete statistical equilibrium has its greatest possible value (for a given energy of the system).

Finally, we may mention another interesting interpretation of the function $S = S(E)$, the entropy of any subsystem or closed system; in the latter case it is assumed that the system is in complete equilibrium, so that its entropy may be expressed as a function of its total energy alone. The statistical weight $\Delta \Gamma = e^{S(E)}$, by definition, is the number of energy levels in the interval $\Delta E$ which describes in a certain way the width of the energy probability distribution. Dividing $\Delta E$ by $\Delta \Gamma$, we obtain the mean separation between adjoining levels in this interval (near the energy $E$) of the energy spectrum of the system considered. Denoting this distance by $D(E)$, we can write

$$D(E) = \Delta E \cdot e^{-S(E)}.$$ (7.18)

Thus the function $S(E)$ determines the density of levels in the energy spectrum of a macroscopic system. Since the entropy is additive, we can say that the mean separations between the levels of a macroscopic body decrease exponentially with increasing size of the body (i.e. with increasing number of particles in it).

From Landau & Lifshitz: Principles of Statistical Physics

§8. The law of increase of entropy

If a closed system is not in a state of statistical equilibrium, its macroscopic state will vary in time, until ultimately the system reaches a state of complete equilibrium. If each macroscopic state of the system is described by the distribution of energy between the various subsystems, we can say that the sequence of states successively traversed by the system corresponds to more and more probable distributions of energy. This increase in probability is in general very considerable, because it is exponential, as shown in §7. We have seen that the probability is given by $e^{S}$, the exponent being an additive quantity, the entropy of the system. We can therefore say that the processes occurring in a non-equilibrium closed system do so in such a way that the system continually passes from states of lower to those of higher entropy until finally the entropy reaches the maximum possible value, corresponding to complete statistical equilibrium.

Thus, if a closed system is at some instant in a non-equilibrium macroscopic state, the most probable consequence at later instants is a steady increase in the entropy of the system. This is the law of increase of entropy or second law.
of thermodynamics, discovered by Clausius; its statistical explanation was given by Boltzmann.

In speaking of the "most probable" consequence, we must remember that in reality the probability of transition to states of higher entropy is so enormous in comparison with that of any appreciable decrease in entropy that in practice the latter can never be observed in Nature. Ignoring decreases in entropy due to negligible fluctuations, we can therefore formulate the law of increase of entropy as follows: if at some instant the entropy of a closed system does not have its maximum value, then at subsequent instants the entropy will not decrease; it will increase or at least remain constant.

There is no doubt that the foregoing simple formulations accord with reality; they are confirmed by all our everyday observations. But when we consider more closely the problem of the physical nature and origin of these laws of behaviour, substantial difficulties arise, which to some extent have not yet been overcome.

Firstly, if we attempt to apply statistical physics to the entire Universe, regarded as a single closed system, we immediately encounter a glaring contradiction between theory and experiment. According to the results of statistics, the universe ought to be in a state of complete statistical equilibrium. More precisely, any finite region of it, however large, should have a finite relaxation time and should be in equilibrium. Everyday experience shows us, however, that the properties of Nature bear no resemblance to those of an equilibrium system; and astronomical results show that the same is true throughout the vast region of the Universe accessible to our observation.

We might try to overcome this contradiction by supposing that the part of the Universe which we observe is just some huge fluctuation in a system which is in equilibrium as a whole. The fact that we have been able to observe this huge fluctuation might be explained by supposing that the existence of such a fluctuation is a necessary condition for the existence of an observer (a condition for the occurrence of biological evolution). This argument, however, is easily disproved, since a fluctuation within, say, the volume of the solar system only would be very much more probable, and would be sufficient to allow the existence of an observer.

The escape from this contradiction is to be sought in the general theory of relativity. The reason is that, when large regions of the Universe are considered, the gravitational fields present become important. According to the general theory of relativity, these fields are just a change in the space-time metric, described by the metric tensor $g_{ik}$. When the statistical properties of bodies are discussed, the metric properties of space-time may in a sense be regarded as "external conditions" to which the bodies are subject. The statement that a closed system must, over a sufficiently long time, reach a state of equilibrium, applies of course only to a system in steady external conditions. The metric
tensor $g_{ik}$ is in general a function not only of the co-ordinates but also of time, so that the "external conditions" are by no means steady in this case. Here it is important that the gravitational field cannot itself be included in a closed system, since the conservation laws which are, as we have seen, the foundation of statistical physics would then reduce to identities. For this reason, in the general theory of relativity, the Universe as a whole must be regarded not as a closed system but as a system in a variable gravitational field. Consequently the application of the law of increase of entropy does not prove that statistical equilibrium must necessarily exist.

Thus this aspect of the problem of the Universe as a whole indicates the physical basis of the apparent contradictions. There are, however, other difficulties in understanding the physical nature of the law of increase of entropy.

Classical mechanics itself is entirely symmetrical with respect to the two directions of time. The equations of mechanics remain unaltered when the time $t$ is replaced by $-t$; if these equations allow any particular motion, they will therefore allow the reverse motion, in which the mechanical system passes through the same configurations in the reverse order. This symmetry must naturally be preserved in a statistics based on classical mechanics. Hence, if any particular process is possible which is accompanied by an increase in the entropy of a closed macroscopic system, the reverse process must also be possible, in which the entropy of the system decreases. The formulation of the law of increase of entropy given above does not itself contradict this symmetry, since it refers only to the most probable consequence of a macroscopically described state. In other words, if some non-equilibrium macroscopic state is given, the law of increase of entropy asserts only that, out of all the microscopic states which meet the given macroscopic description, the great majority lead to an increase of entropy at subsequent instants.

A contradiction arises, however, if we look at another aspect of the problem. In formulating the law of increase of entropy, we have referred to the most probable consequence of a macroscopic state given at some instant. But this state must itself have resulted from some other states by means of processes occurring in Nature. The symmetry with respect to the two directions of time means that, in any macroscopic state arbitrarily selected at some instant $t = t_0$, we can say not only that much the most probable consequence at $t > t_0$ is an increase in entropy, but also that much the most probable origin of the state was from states of greater entropy; that is, the presence of a minimum of entropy as a function of time at the arbitrarily chosen instant $t = t_0$ is much the most probable.

This assertion, of course, is not at all equivalent to the law of increase of entropy, according to which the entropy never decreases (apart from entirely negligible fluctuations) in any closed systems which actually occur in Nature. And it is precisely this general formulation of the law of increase of entropy
which is confirmed by all natural phenomena. It must be emphasised that it is certainly not equivalent to the formulation given at the beginning of this section, as it might appear to be. In order to derive one formulation from the other, it would be necessary to use the concept of an observer who artificially “creates” a closed system at some instant, so that the problem of its previous behaviour does not arise. Such a dependence of the laws of physics on the nature of an observer is quite inadmissible, of course.

At the present time it is not certain whether the law of increase of entropy thus formulated can be derived on the basis of classical mechanics. It may be noted that, because the equations of classical mechanics are invariant under time reversal, we can consider only the deduction that the entropy varies monotonically. In order to derive a law of monotonic increase, we should have to define the future as the direction of time in which the entropy increases, and the problem would then arise of proving that this definition of the future and the past is the same as the definition used in quantum mechanics (see below).

It is more reasonable to suppose that the law of increase of entropy in the above general formulation arises from quantum effects.

The fundamental equation of quantum mechanics, namely SCHRÖDINGER's equation, is itself symmetrical under time reversal, provided that the wave function $Ψ$ is also replaced by $Ψ^*$. This means that, if at some instant $t = t_1$ the wave function $Ψ = Ψ(t_1)$ is given, and if according to SCHRÖDINGER's equation it should become $Ψ(t_2)$ at some other instant $t_2$, then the change from $Ψ(t_1)$ to $Ψ(t_2)$ is reversible; in other words, if $Ψ = Ψ^*(t_2)$ at the initial instant $t_1$, then $Ψ = Ψ^*(t_1)$ at $t_2$.

However, despite this symmetry, quantum mechanics does in fact involve an important non-equivalence of the two directions of time. This appears in connection with the interaction of a quantum object with a system which with sufficient accuracy obeys the laws of classical mechanics, a process of fundamental significance in quantum mechanics. If two interactions $A$ and $B$ with a given quantum object occur in succession, then the statement that the probability of any particular result of process $B$ is determined by the result of process $A$ can be valid only if process $A$ occurred earlier than process $B$.

Thus in quantum mechanics there is a physical non-equivalence of the two directions of time, and the “macroscopic” expression of this may in fact be the law of increase of entropy. Up to the present, however, this relation has not been at all convincingly shown to exist in reality. If this is indeed the origin of the law of increase of entropy, there must exist an inequality involving the quantum constant $ℏ$ which ensures the validity of the law and is satisfied in the real world (and probably satisfied by a very wide margin).

Summarising, we may repeat the general formulation of the law of increase of entropy: in all closed systems which occur in Nature, the entropy never

† See also Quantum Mechanics, §7.
decreases; it increases, or at least remains constant. In accordance with these two possibilities, all processes involving macroscopic bodies are customarily divided into irreversible and reversible processes. The former comprise those which are accompanied by an increase of entropy of the whole closed system; the reverse processes cannot occur, since the entropy would then have to decrease. Reversible processes are those in which the entropy of the closed system remains constant\(^\dagger\), and which can therefore take place in the reverse direction. A strictly reversible process is, of course, an ideal limiting case; processes actually occurring in Nature can be reversible only to within a certain degree of approximation.

\(^\dagger\) It must be emphasised that the entropies of the individual parts of the system need not remain constant also.